Unit 2: Geometrical properties of individual features

1: Introduction
2: Individual properties (geometry)
3: Spatial pattern (relationships)
1 Introduction
Description of spatial properties (1)

Production of indices describing spatial properties

- Individual spatial properties:
  - Geometry: location, size, shape

- Spatial arrangement of features (pattern):
  - Spatial relationships: distribution, neighborhood, proximity

- Distinction between object mode and image mode:
  - object mode: units of observation are features (object)
  - image mode: regions are features

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Description of spatial properties (2)

Described spatial features are models of those from the reality

- Their geometry is simplified:
  - in object mode: through the use of geometrical primitives
  - in image mode: through the choice of a resolution

- Their attributes corresponds to a global thematic property:
  - in object mode: point, linear, areal units
  - in image mode: the set of areal units (cells) composing the region
Description of spatial properties (3)

Descriptors summarize some properties of spatial features

- They are indicators (indices):
  - expressing some properties, not all
  - dependent on the quality of the modeled features

- Such indicators should be considered as estimators of properties:
  - several indicators can express the same property
  - their use and interpretation should be made with sound understanding
2

Individual properties of point features
Point features: individual properties

- A point feature is modeled as a geometrical point, it has no spatial dimension (0D)
- In image mode the point region is also considered as spatially dimensionless, despite the fact it is made of an areal spatial unit (the cell)
- Its single individual geometric property is:
  - its location
Point object: Location (position)

- **Horizontal (X coordinate) and vertical (Y coordinate) positions in the projected plane, using a defined coordinate system**

- **Example:**
  Location of object $i$: $(251.18\,\text{m}, 139.54\,\text{m})$

\[
\begin{array}{c}
\text{Coordinate X} \\
245 \ 255 \ 265 \\
\text{Coordinate Y} \\
147 \ 137 \ 127 \\
\end{array}
\]

\[
\begin{align*}
X_i &= 251.18 \\
Y_i &= 139.54
\end{align*}
\]
Point region: Location (position)

- **Horizontal and vertical position:**
  - located at center of the cell (X,Y coordinates)
  - corresponding to the cell position in the grid (column and row)
  - grid resolution dependent

- **Example:**
  Location of region i:
  (251.5m, 139.5m), with a grid resolution of 1m
  or (7,8) in grid coordinates (column, row)

\[
X_i = 251.5 \quad Y_i = 139.5
\]

\[
\text{or} \quad C_i = 7 \quad R_i = 8
\]
Individual properties of linear features
Linear feature: nature and type

- A linear feature is modeled as a geometrical **broken line** or a **chain**, it has one spatial dimension (**1D**).
- In image mode a **linear region** is a set of contiguous cells having only one spatial dimension too.
- A linear feature can be:
  - **simple**: made of a single chain
  - **complex**: made of several chains

**A network** can be considered either as:
- a single feature (complex linear feature)
- a group of numerous features with **connections**
Linear feature: simple and complex

Simple linear object
- Object made of a single chain ($C_1$)
- Example: a segment

Complex linear object
- Object made of several chains ($C_1, \ldots, C_n$)
- Example: a set of segments, a whole network

- vertex
- nod
Linear feature: individual geometric properties

- Only **simple linear feature** will be discussed here
- Set of linear features such as network will be discussed in Lesson « Network description »
- Individual geometric properties of a linear feature are:
  - its **location** (position)
  - its **size** (length)
  - its **shape** (sinuosity)
  - its **orientation** (direction)
Linear object: Location

- Generally its location is considered as the horizontal and vertical position into the project plane of its **Mean center** (MC) or so called **Gravity center**

- **Example:**

<table>
<thead>
<tr>
<th>Point</th>
<th>X</th>
<th>Y</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>247</td>
<td>133.5</td>
</tr>
<tr>
<td>2</td>
<td>251.5</td>
<td>136</td>
</tr>
<tr>
<td>3</td>
<td>256</td>
<td>133</td>
</tr>
<tr>
<td>4</td>
<td>263</td>
<td>137</td>
</tr>
<tr>
<td>∑</td>
<td>1017.5</td>
<td>539.5</td>
</tr>
<tr>
<td>MC</td>
<td>254.4</td>
<td>134.9</td>
</tr>
</tbody>
</table>

\[ \bar{X} = 254.4 \quad \bar{Y} = 134.9 \]
Linear object: Size (length)

- It is the sum of length of the \( n \) segments composing the chain:
  \[
  L = \sum D_{s_i}
  \]
  with: \( D_{s_i} = \) distance between the two segment \( i \) ends (vertices)

- **Exemple:**

<table>
<thead>
<tr>
<th>Segment</th>
<th>( \Delta x )</th>
<th>( \Delta y )</th>
<th>( \Delta x^2 )</th>
<th>( \Delta y^2 )</th>
<th>( \Delta x^2 + \Delta y^2 )</th>
<th>( D_{s_i} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4.5</td>
<td>2.5</td>
<td>20.25</td>
<td>6.25</td>
<td>26.5</td>
<td>5.15</td>
</tr>
<tr>
<td>2</td>
<td>4.5</td>
<td>3</td>
<td>20.25</td>
<td>9</td>
<td>29.25</td>
<td>5.41</td>
</tr>
<tr>
<td>3</td>
<td>7</td>
<td>4</td>
<td>49</td>
<td>16</td>
<td>65</td>
<td>8.06</td>
</tr>
</tbody>
</table>

\[
\sum s_i = 18.62
\]

\[
L = 5.15 + 5.41 + 8.06 = 18.62 \text{m}
\]
Linear region: Size (length)

- It is the sum of length of the n units (cells) composing the region:

  This metric uses 2 yardsticks:
  - diagonal = 1.41 unit, side = 1 unit

- Example:
  Let a grid with 1m resolution:
  \[ L = 1.41 + 1.41 + 1 + 1 + 1.41 + 1 + 1.41 + 1.41 + 1 + 1.41 + 1 + 1.41 + 1.41 \]
  \[ L = ((10 \times 1.41) + (5 \times 1)) \times 1m = 19.1m \]

In image mode the estimation of length is systematically exaggerated (see the estimation in object mode: \( L = 18.62m \))
Linear object: Shape (sinuosity)

- It is the ratio between the chain length $L$ and the distance $D_{df}$ between its two ends:
  \[ S = \frac{L}{D_{df}} \]

- Interpretation:
  $S$ is a ratio, with $S \geq 1$

- Example:

<table>
<thead>
<tr>
<th>$\Delta x$</th>
<th>$\Delta y$</th>
<th>$\Delta x^2$</th>
<th>$\Delta y^2$</th>
<th>$\Delta x^2 + \Delta y^2$</th>
<th>$D_{df}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>16</td>
<td>3.5</td>
<td>256</td>
<td>12.25</td>
<td>268.25</td>
<td>16.38</td>
</tr>
</tbody>
</table>

$L = 5.15 + 5.41 + 8.06 = 18.62$

$D_{df} = 16.38$

$S = 18.62 / 16.38 = 1.14$
Linear region: Shape (sinuosity)

- It is the ratio between the region length $L$ and the distance $D_{df}$ between its 2 ends:
  \[ S = \frac{L}{D_{df}} \]

- Interpretation:
  $S$ is a ratio, with $S \geq 1$

- Example:
  \[
  L = ((10 \times 1.41) + (5 \times 1)) \times 1\text{m} = 19.1\text{m}
  \]
  \[
  D_{df} = ((4 \times 1.41) + (11 \times 1)) \times 1\text{m} = 16.64\text{m}
  \]
  \[ S = \frac{19.1}{16.64} = 1.15 \]

As these 2 terms $L$ and $D_{df}$ use the same metric, the resulting value $S$ is close to the one obtained in object mode ($S = 1.14$)
Linear object: Orientation (direction)

- It is the angle of the main direction of the chain with respect to the vertical the main direction and the vertical cross at the Mean center (MC)

- **Example:**
  
  Chain direction:
  
  $\tau = 56^\circ$

  It is the same chain as in previous illustration, except its direction is different
Individual properties of areal features
Areal feature: nature and type

- An areal object is modeled as a geometrical closed chain (polygon), it has 2 spatial dimensions (2D)
- In image mode the areal region is made of a set of contiguous cells having 2 spatial dimensions
- An areal feature can be:
  - simple: made of a single polygon or region
  - complex: made of several polygons (with inner or outer area: island)
Areal object: simple and complex

**Simple areal object**
- Object made of a single polygon (closed chain)

**Complex areal object**
- Object made of several polygons (closed chains)
- **Example**: object with inner and outer area (islands)

**Convex**
- chain $C_1$

**Concave**
- chain $C_1$

- vertex
- nod

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Areal feature: individual properties

- Only **simple areal features** will be discussed here
- Individual geometric properties of an areal feature are:
  - its **location** (position)
  - its **size** (perimeter, area)
  - its **shape** (compactness)
Objet zonal: Location (position)

- Horizontal and vertical position in the projected plane of its **Mean center (MC)**, or so called **gravity center**

- Example:

<table>
<thead>
<tr>
<th>Point</th>
<th>X</th>
<th>Y</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>248</td>
<td>138</td>
</tr>
<tr>
<td>2</td>
<td>252</td>
<td>143</td>
</tr>
<tr>
<td>3</td>
<td>262</td>
<td>142</td>
</tr>
<tr>
<td>4</td>
<td>263</td>
<td>133</td>
</tr>
<tr>
<td>5</td>
<td>255</td>
<td>132</td>
</tr>
<tr>
<td>Σ</td>
<td>1280</td>
<td>688</td>
</tr>
<tr>
<td>CM</td>
<td>256</td>
<td>137.6</td>
</tr>
</tbody>
</table>

\[
\bar{X} = 256 \quad \bar{Y} = 137.6
\]

Coordinate X

Coordinate Y

Example:
Areal object: Size (perimeter)

- It is the sum of length of the n segments composing the chain:
  \[ L = \sum D_{s_i} \]
  with: \( D_{s_i} \) = distance between the two segment i ends (vertices)

- Example:

<table>
<thead>
<tr>
<th>Segment</th>
<th>( \Delta x )</th>
<th>( \Delta y )</th>
<th>( \Delta x^2 )</th>
<th>( \Delta y^2 )</th>
<th>( \Delta x^2 + \Delta y^2 )</th>
<th>( D_{s_i} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4</td>
<td>5</td>
<td>16</td>
<td>25</td>
<td>41</td>
<td>6.4</td>
</tr>
<tr>
<td>2</td>
<td>10</td>
<td>1</td>
<td>100</td>
<td>1</td>
<td>101</td>
<td>10</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>9</td>
<td>1</td>
<td>81</td>
<td>82</td>
<td>9</td>
</tr>
<tr>
<td>4</td>
<td>8</td>
<td>1</td>
<td>64</td>
<td>1</td>
<td>65</td>
<td>8.1</td>
</tr>
<tr>
<td>5</td>
<td>7</td>
<td>6</td>
<td>49</td>
<td>36</td>
<td>85</td>
<td>9.2</td>
</tr>
</tbody>
</table>

\( \sum s_i = 42.7 \)

\[ L = 6.4 + 10 + 9 + 8.1 + 9.2 = 42.7 \text{m} \]
Areal region: Size (perimeter)

The perimeter can be evaluated with 2 different techniques:

- **External perimeter of the region (envelop)**:
  - staircase effect using “Manhattan distance”
  - systematic over estimation of the perimeter

- **Length of the linear region edge (linear perimeter)**:
  - reduced staircase effect by taking the diagonal distance into account
  - under estimation of the perimeter with a too coarse resolution

*These two estimation are dependant on the metric used and the grid resolution*
Areal region: Size (perimeter of the envelop)

- **It is the sum of length of the n cell sides bounding the region:**
  
  This metric distance is often called Manhattan yardstick: side = 1 unit

- **Example:**
  
  Let a grid with 1m resolution (unit):

  \[ L = (50*1) \times 1m = 50m \]

  In image mode, the use of this metric tends to over estimate the measure of perimeter (see estimation in object mode: \( L = 42.7m \))
Areal region: Size (linear perimeter)

- It is the sum of length of the n cells bounding the region:
  - This metric uses 2 yardsticks: diagonal = 1.41 unit, side = 1 unit

- Example:
  - Let a grid with 1m resolution (unit):
    \[ L = 1.41 + 1.41 + 1 + 1.41 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1.41 + 1 + 1 + 1.41 + 1 + 1 + 1.41 + 1.41 + 1 + 1 + 1 + 1 + 1.41 + 1 + 1 + 1.41 + 1.41 + 1 + 1.41 + 1.41 \]
    \[ L = ((11 \times 1.41) + (24 \times 1)) \times 1m = 39.51m \]

The estimation of perimeter using this technique is close to the one in object mode (L = 42.7m)
Areal object: Size (area)

There are several techniques to estimate the area of an object:

- For **non generalized** features (eg. features manually delineated on a map or an image):
  - random or regular **point sampling** technique (Unwin D., 1981, p.126)
  - assuming objects are already generalized into polygons, this technique will not be discussed here

- For **areal objects** **numerically described with polygons** (for a GDB in object mode):
  - by breaking up into triangles
  - by breaking up into rectangles (computer algorithm)
Areal object: Size (area) - split into triangles

- The area of polygon $A_p$ is the sum of areas of its composing triangles $A_{ti}$:
  \[ A_p = \sum A_{ti} \]
  with: $A_{ti} = \frac{B \times h}{2}$, 
  B the basis and h the height

- Example:

<table>
<thead>
<tr>
<th>Triangle</th>
<th>Basis</th>
<th>Height</th>
<th>B x h</th>
<th>$A_{ti}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>14.5</td>
<td>3.7</td>
<td>53.65</td>
<td>26.82</td>
</tr>
<tr>
<td>2</td>
<td>14.5</td>
<td>7.7</td>
<td>111.65</td>
<td>55.82</td>
</tr>
<tr>
<td>3</td>
<td>12.3</td>
<td>6</td>
<td>73.8</td>
<td>36.9</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>\sum $A_{ti}$</td>
<td>119.54</td>
</tr>
</tbody>
</table>

  $A_p = 26.82 + 55.82 + 36.9 = 119.54 \text{m}^2$

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Areal object: Size (area) - split into rectangles

This technique is adapted for a numerical vector structure (GDB)

Assuming polygon vertices are sequentially described clockwise from a given starting point:

\[ A = 0.5 \times \sum y_i (x_{i+1} - x_{i-1}) \]

- **Example:**

<table>
<thead>
<tr>
<th>X_i</th>
<th>Y_i</th>
<th>Contribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>248</td>
<td>138</td>
<td>138(252-255) = -414</td>
</tr>
<tr>
<td>252</td>
<td>143</td>
<td>143(262-248) = 2002</td>
</tr>
<tr>
<td>262</td>
<td>142</td>
<td>142(263-252) = 1562</td>
</tr>
<tr>
<td>263</td>
<td>133</td>
<td>133(255-262) = -931</td>
</tr>
<tr>
<td>255</td>
<td>132</td>
<td>132(248-263) = -1980</td>
</tr>
</tbody>
</table>

\[ \sum \text{contributions} = 239 \]

\[ A = 0.5 \times 239 = 119.5 \text{m}^2 \]
Areal region: Size (area)

- It is the sum of areas of cells composing the region:
  \[ A = n \times A_u \]
  with \( A_u \) constant for the \( n \) units

- **Example:**
  Let a grid with \( 1 \text{m}^2 \) resolution (unité):
  \[ A = 110 \times 1\text{m}^2 = 110\text{m}^2 \]

In image mode, estimation of the area is close to the one in object mode: \( A = 119.5\text{m}^2 \)
Areal feature: Shape (indices)

The shape of areal features is a very rich concept that is difficult to summarize with a single index.

Such indices should allow the comparison between features:

- independent of the description scale and the size of features
- with a reference to a particular shape

This index should be a ratio with at least one reference value.
Areal feature: Shape (compactness indices)

Among the numerous indices proposed in the literature, those describing the compactness of the shape

- **Counter-example:** perimeter / area index (P/A)
  - it is simple to produce (based on size indices)
  - but its value is dependant on the unit of measurement as well as on the size of features. its use is therefore strongly limited for the comparison of features compactness

- A compactness index refers to a geometrically compact shape, such as a circle or sometimes a square
Basic elements for compactness indices

- **For the concerned feature:**
  A : area of the feature
  L : major axis (distance between the 2 most faraway vertices of the feature)

- **For the reference feature (circle):**
  C : area of smallest circumscribing circle
  R_C: radius of smallest circumscribing circle
  I : area of largest inscribed circle
  R_I: radius of largest inscribed circle

- **Example:**
  A = 119.5m²  L = D_{1,4} = 15.8m
  R_C= 8m  C = \pi R^2 = 201.06m²
  R_I= 5.1m  I = \pi R^2 = 81.71m²
Areal feature: Compactness indices

Most usual compactness indices are made of:

- The ratio between the feature area and the area of its smallest circumscribing circle:
  \[ S_{A,C} = \frac{A}{C} \]

- The ratio between the feature area and the area of a circle having the major axis length \( L \) as perimeter:
  \[ S_{A,L} = \frac{A}{\pi (0.5 L)^2} = 1.27 \frac{A}{L^2} \]

- The ratio between the largest inscribed circle area and the area of its smallest circumscribing circle:
  \[ S_{I,C} = \frac{I}{C} \]
Areal feature: Compactness indices (continued)

Some other indices derived:

- From the ratio between the feature area and the area of its smallest circumscribing circle:
  
  \[ S'_{A,C} = \sqrt{\frac{A}{C}} \quad \text{Sr}_{A,C} = \frac{R_A}{R_C}, \text{ with } R_A = \sqrt{\frac{A}{\pi}} \]

- From the ratio between the largest inscribed circle area and the area of its smallest circumscribing circle:
  
  \[ \text{Sr}_{I,C} = \frac{R_I}{R_C} \]

- From the ratio between the minor and the major axis:
  
  \[ S_{I,L} = \frac{l}{L} \quad \text{with } l \text{ being the minor axis, perpendicular to the major axis} \]
Areal feature: Compactness indices (continued)

And some easily computable indices

- Some basic elements involved in the computation of compactness are difficult or tedious to produce for irregular features:
  - particularly inscribed and circumscribing radius
- In numerous GIS software proposed compactness indices are therefore computed as follow:
  - \( S_{A,C_p} = \frac{A}{C_p} \), with \( C_p \) as the area of a circle having the same perimeter as the feature
  - \( S_{A,Q} = \frac{A}{Q} \), with \( Q \) as the area of a circumscribing square with a side length equal to \( L \)
Areal feature: Compactness indices - Comparison

<table>
<thead>
<tr>
<th>Indices</th>
<th>Formula</th>
<th>Circle</th>
<th>Oriented square</th>
<th>Irregular polygon</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_{A,C}$</td>
<td>$= A/C$</td>
<td>$= 1$</td>
<td>$= 0.64$</td>
<td>$= 119.5 / 201.06 = 0.59$</td>
</tr>
<tr>
<td>$S_{A,L}$</td>
<td>$= 1.27 A/L^2$</td>
<td>$= 1$</td>
<td>$= 0.64$</td>
<td>$= 119.5 / 196.07 = 0.61$</td>
</tr>
<tr>
<td>$S_{L,C}$</td>
<td>$= L/C$</td>
<td>$= 1$</td>
<td>$= 0.5$</td>
<td>$= 81.7 / 201.6 = 0.41$</td>
</tr>
<tr>
<td>$S'_{A,C}$</td>
<td>$= \sqrt{(A/C)}$</td>
<td>$= 1$</td>
<td>$= 0.8$</td>
<td>$= (119.5 / 201.06)^{0.5} = 0.77$</td>
</tr>
<tr>
<td>$S_{rA,C}$</td>
<td>$= R_i/R_c$</td>
<td>$= 1$</td>
<td>$= 0.71$</td>
<td>$= 5.1 / 8 = 0.64$</td>
</tr>
<tr>
<td>$S_{L,L}$</td>
<td>$= L/L$</td>
<td>$= 1$</td>
<td>$= 1$</td>
<td>$= 10.9 / 15.8 = 0.69$</td>
</tr>
<tr>
<td>$S_{A,CP}$</td>
<td>$= A/C_P$</td>
<td>$= 1$</td>
<td>$= 0.71$</td>
<td>$= 119.5 / 144.3 = 0.83$</td>
</tr>
<tr>
<td>$S_{A,Q}$</td>
<td>$= A/Q$</td>
<td>$= 0.78$</td>
<td>$= 0.5$</td>
<td>$= 119.5 / 15.8^2 = 0.48$</td>
</tr>
</tbody>
</table>

Characteristics of the irregular polygon (illustration of the areal object):
$A = 119.5$, $L = 15.8$, $l = 10.9$, $P = 42.7$, $C= 201.06$, $R_c = 8$, $R_i = 5.1$
Areal feature: Compactness indices - Comments

All these indices express the relative compactness of a feature with respect to a compact shape of reference:

- For all except the last index, the reference is a circular shape:
  - the maximum value 1 expresses a maximal compactness
  - the lesser the compactness of the polygon, the lower the index value

- Each index expresses differently the discrepancy between the feature shape and the reference shape:
  - It is therefore important to master the meaning of index values.
Areal feature: Compactness indices - References

Suggested references

Baker L., :
Davis P., :
Ebdon D., :
Fitzgerald B., :
Hammond, Mc Cullagh, :
Unwin D., :
Idrisi (Cratio) :
ArcGIS :
Arrangement spatial des objets ponctuels
B-AN / L2
Discrete spatial variables

U2: Spatial properties of features

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