Geographic Information Technology Training Alliance (GITTA) presents:

Terrain analysis (intermediate)

Responsible persons: Eric Lorup, Helmut Flitter, Marcel Frehner, Robert Weibel, Samuel Wiesmann

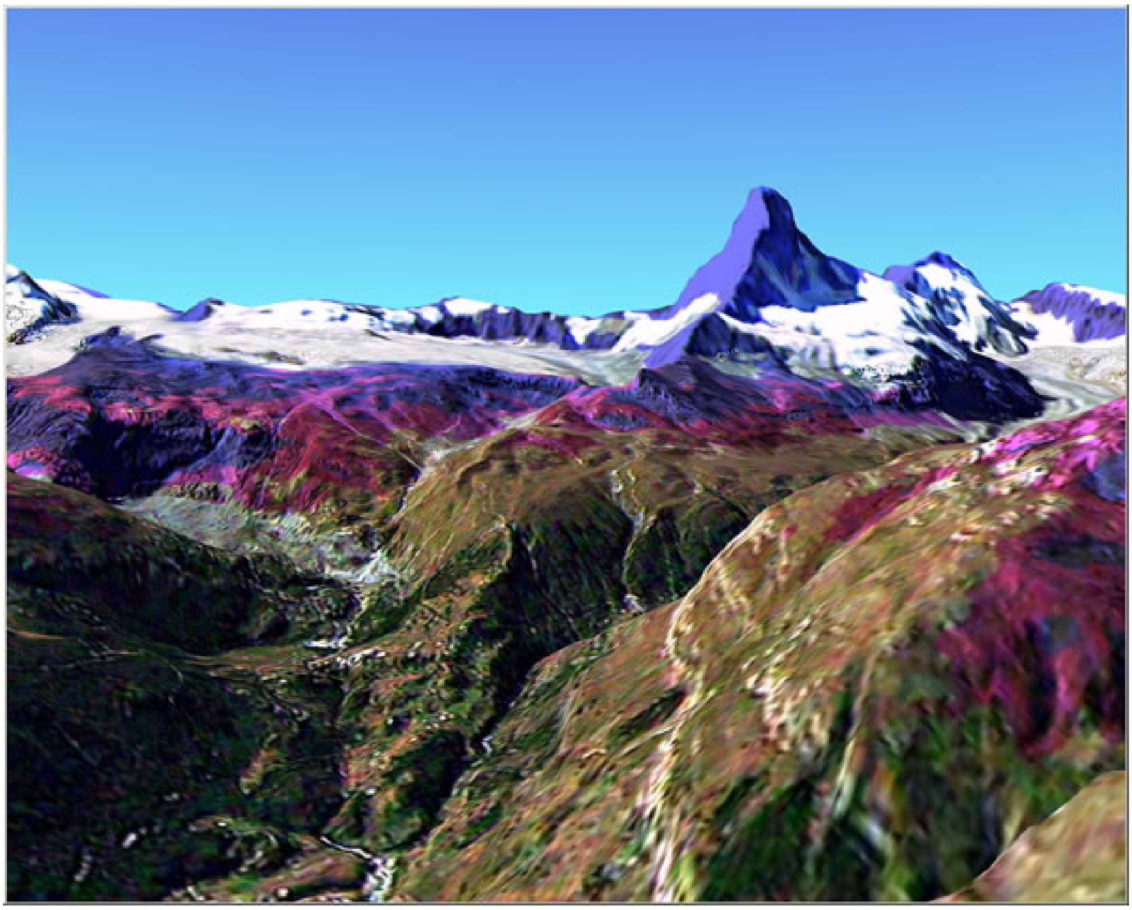
Table Of Content

1. Terrain analysis (intermediate) 2
1.1. Applications in hydrology 5
1.1.1. Grids vs. TIN’s 6
1.1.2. Drainage networks 6
1.1.3. Issues and alternative approaches 9
1.1.4. Information derived from a drainage network 12
1.1.5. Questions 17
1.1.6. Multiple choice quiz 17
1.1.7. Summary 19
1.2. Visibility analysis and related topics 20
1.2.1. External effects 20
1.2.2. Line-of-sight problem 24
1.2.3. Moving objects 27
1.2.4. Watchtower problem 27
1.2.5. Maps of topographic shadows 29
1.2.6. Horizon lines 31
1.2.7. Potential direct solar radiation 31
1.2.8. Exercise curvature and atmospheric refraction 32
1.2.9. Exercise atmospheric interferences 32
1.2.10. Multiple choice quiz 33
1.2.11. Summary 34
1.3. Bibliography 36

1. Terrain analysis (intermediate)

Overview of information products derivable from DTM's

Information products such as elevation, slope, aspect, profile curvature and plan curvature can quite easily be derived from digital terrain models (DTM) (see basic lesson **Geländeanalyse**). Sometimes these basic terrain parameters are not sufficient and more complex information is required in order to make well-founded decisions. Imagine somebody wants to model permafrost in the Alps. He/she would need information about slope and aspect, which is quite easy to derive from a DTM. Topographic shadows are possibly a little more demanding but having received this information it can be used for modelling potential solar radiation. This again can be employed in an even more sophisticated permafrost computer model. The following illustration from Gruber et al. (2001) shows the result of a permafrost model:



Permafrost model in the Matterhorn region (Gruber et al. 2001)

This illustration is based on a multiple regression with the potential direct shortwave radiation in summer and the sea level is the independent variable; the BTS (basis temperature of snow-cover in late winter) is the dependent variable. Ca. 450 measurements have been used as a basis for the model. Violet means possible permafrost and blue means likely permafrost. To better demonstrate the context, the authors fused satellite images with the permafrost classes. Computergraphics by **Stefan Biegger**. Data source: Swisstopo (1991). Satellite images: © **ESA/Eurimage**, **CNES/Spotimage**, **NPOC**.

So in this lesson we are proceeding from primary information products like slope, aspect and curvature towards more realistic applications in the realm of hydrology (unit **Applicatons in hydrology**) and visibility analysis (unit **Visibility analysis**). Table 1 lists some information products derivable from DTM's. The table is subdivided into primary and secondary information. Primary information can be derived quite directly from DTM's and is used universally. Secondary information only results after several processing steps and its application is more specific. Always keep in mind that the type of terrain model utilised (raster/vector, high/ low resolution, photogrammetric/radar…) has to suit the respective application. An engineer planning a road through a mountainous area might use a very precise TIN whereas a low-resolution grid might be adequate for a vegetation model in a flat area.

	Comment	**Possible fields of application**
Primary topographic information		
Elevation		Universal
Slope	Angle of steepest descent [0-90°]	Universal
Aspect	Angle between north and direction of steepest descent [0-360°]	Universal
Curvature (plan, profile)	[Convex-concave]	Universal
Secondary topographic information		
Local drain direction net (ldd net)	Topology of water drainage in a grid. Eight directions for each cell.	Hydrology
Upstream elements, catchment area, flow accumulation	Number of cells upstream of a given cell.	Hydrology
Catchment length	Distance from highest point to outlet	Hydrology
Wetness index	Measure of wetness	Hydrology
Stream power index	Measure of the erosive power of overland flow	Hydrology
Sediment transport index	Characterisation of erosion and deposition processes (compare universal soil loss equation)	Hydrology
Stream length	Length of longest path along ldd upstream of a given cell	Hydrology
Stream channel	Cells with a certain minimum number of upstream elements	Hydrology
Ridge	Cells with no upstream elements	Hydrology
Intervisibility, viewshed	Visible/invisible area	Positioning of radio antennas
Cast shadow	Shadows cast by nearby terrain	Radiation modelling

Horizon line	Line between farthest points on the terrain still visible from a given point	Engineering, heuristic for visibility analysis
Potential direct solar radiation	Solar radiation received at a location in the terrain depending on the incoming solar radiation, the aspect and slope of the terrain and shadows cast by terrain nearby	Engineering, vegetation modelling, permafrost modelling

Learning Objectives

- You understand how digital terrain models can be applied in hydrology to derive flow paths, hydrologic indices, and geomorphologic features related to hydrology.
- You know how DTM's can be used in order to derive horizon lines, viewsheds, topographic shadows and maps of potential direct radiation.

1.1. Applications in hydrology

Although there are many influences on an areas water balance, the terrain is a key factor as water usually runs downhill in the direction of the steepest slope. DTM's therefore, are often applied in hydrology specific computer models. Nevertheless we shouldn't forget that water sometimes infiltrates into the soil, evaporates from the soil or is intercepted by plants without flowing downhill. Also the inertia of water may occasionally cause a somewhat atypical behaviour, e.g. it may flow uphill for a limited distance. However DTM's have proved advantageous as a basis for hydrologic modelling. Further information like evaporation, infiltration or interception can always additionally be included into a hydrological model and contribute to an increased plausibility.

Rhinefalls in Schaffhausen: Try to spot some ridges and some flow channels in the picture. (Photo: Sabine Timpf)

As hydrology is closely linked to terrain features, the potential of hydrologic modelling goes beyond pure runoff prediction:

- The extraction of drainage networks sometimes uncovers points and lines without any catchment at all. Those can be identified as ridges.
- Other areas possibly have a very large catchment and can therefore be considered as part of a flow channel.

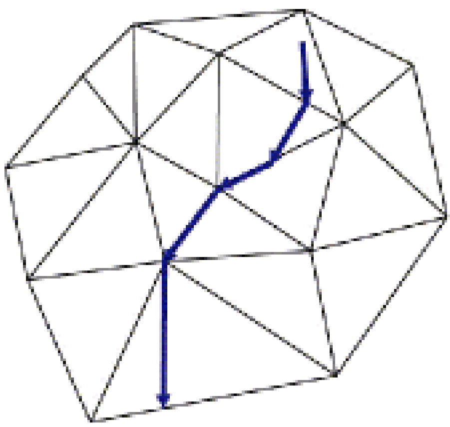
In such a way, hydrologic rules can help to extract geomorphologic information from a DTM.

Learning Objectives

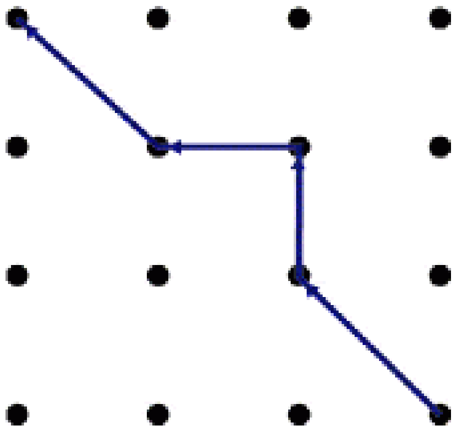
- You know how a local drain direction net (ldd net) can be extracted from a raster DTM.
- You are able to use an ldd net to derive hydrologic indices such as; a wetness index or a stream transport index and you are able to compute topographic features such as ridges or stream channels.

1.1.1. Grids vs. TIN's

The overwhelming part of commercial implementations for hydrologic modelling focuses on gridded DTM's, rather than TIN's. Grid-based algorithms are far simpler than their counterparts for TIN's and gridded DTM's from major data producers are widely available. Therefore, only the derivation of hydrological information from gridded DTM's is discussed here. We assume that each grid point lies in the center of a raster cell, which it can be associated with.
Yet TIN's have a better potential of accurately representing drainage features than grids, as flow directions are arbitrary. Grids have a rigid sampling structure and merely allow 8 flow directions (4 cardinal, 4 diagonal). The channels and ridges are forced to the locations of grid points, although they would rarely pass through them in reality.



TIN's represent drainage features more accurately. Flow directions can be arbitrary.



In grids the flow directions are restricted to the grid points. There are only 8 possible flow directions.

1.1.2. Drainage networks

The basis for the derivation of many hydrologic parameters is a drainage network, which can manually be digitised into a grid or extracted automatically. The latter is usually the case. You will learn how to do this in the following sections. Local drain directions (i.e. flow directions) in grids are often coded according to the numeric pad on a computer keyboard. A cell discharging southwest would therefore be allocated the value 1, a cell discharging west would be allocated the value 4 and so on.

7	8	9
4		6
1	2	3

Local drain directions in a raster can be represented by the numbers on a keyboard.

A map assigning a local drain direction to each grid point is called a local drain direction net or ldd net (Burrough et al. 1998). Various algorithms exist for computing ldd nets, each based on different assumptions on the drainage of water in the terrain.

D8 Algorithm

The D8 algorithm is based on the quite strict assumption, that within a window of 9 cells water always flows in the direction of the steepest slope. I.e. the slope to each of the 8 neighbours of a central source cell has to be computed using the following formula:

$$slope = \arctan\left(\frac{\Delta z}{d}\right)$$

where *d* is the distance and is the elevation difference between two grid points. In a raster, where the cell width is 1 we have to distinguish between two cases:

- If we calculate the slope in the direction of north, east, south, or west, the distance *d* between the grid points is 1.
- If we want to calculate the slope in diagonal directions (northeast, southeast, southwest, northwest), the distance *d* between the grid points is .

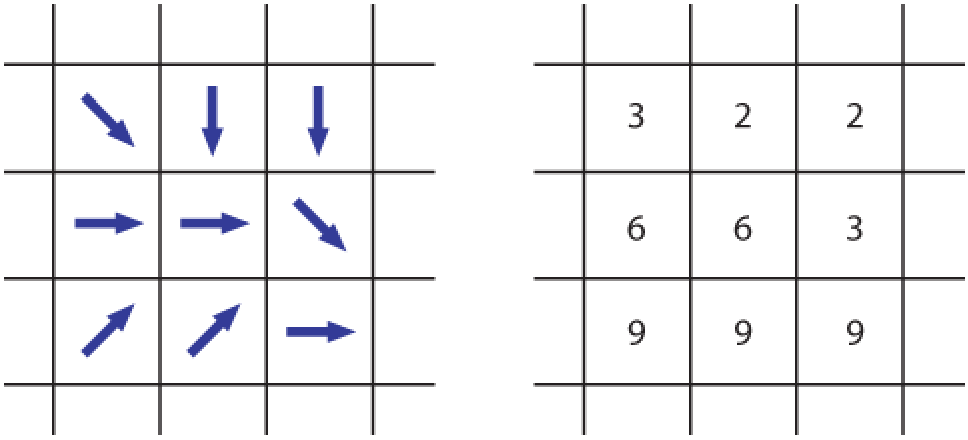
Find the direction of flow

Let's have a look at the following example, where the source cell and its 8 neighbours have defined elevation values and the cell size is 1, 2 or 3. Where does the water drain to? Select the correct cell by clicking on it. (Attention: this little interactive demonstration is not perfect. Where two cells might serve as target cells equally, there is no special solution implemented - the application simply chooses the first of these two cells in a sequence!)

Only pictures can be viewed in this version! For Flash, animations, movies etc. see online version. Only screenshots of animations will be displayed. [link]

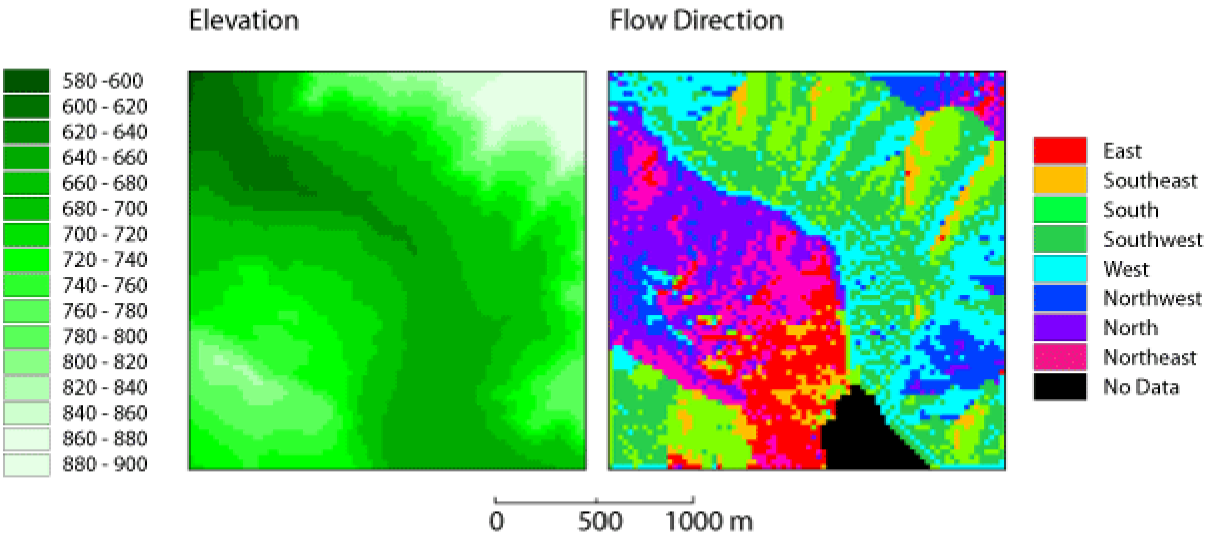
The local drain direction net - ldd net

We now know how to calculate the local drain direction for a single cell in the center of a window of 9 cells. By moving the window stepwise through the entire map we can assign a local drain direction to every cell and get a "local drain direction net" or "ldd net" at the end.



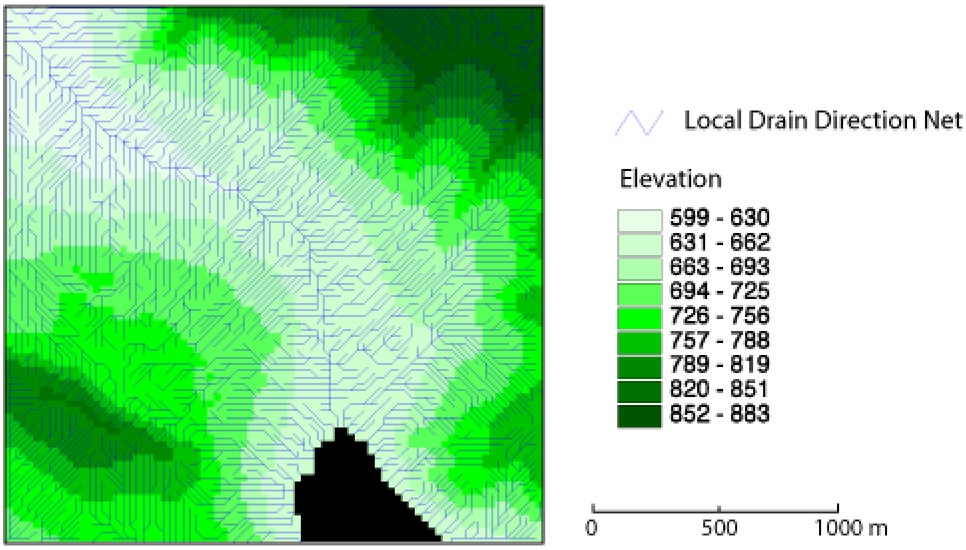
Local drain direction net (ldd net) for a small DTM. Left: Local drain directions coded as arrows. Right: Local drain directions coded according to the numeric pad on a computer keyboard.

The example illustration of the flow directions below shows a 25m raster DHM from the Tuerlersee area and the flow direction map derived from it. The area of the lake in the southern part of the map had to be excluded because the drain direction on a horizontal plain is undefined.



Flow directions derived from a 25m raster DHM by using a D8 algorithm. (Swisstopo 1991)

An ldd net can alternatively be visualised with a vector map as shown in the following image.



An ldd net as a vector map.

1.1.3. Issues and alternative approaches

There are some points to think about when you try to derive an ldd net. The information value of the output often depends on a clever choice of data processing and preprocessing methods some of which are described below. In order to choose wisely the nature of your analysis has to be considered and the DTM metadata have to be inspected carefully. If satisfactory metadata are not available advice from other people who have worked with the same data or who have done similar analyses before can help to find the optimal solution.

Stochastic methods

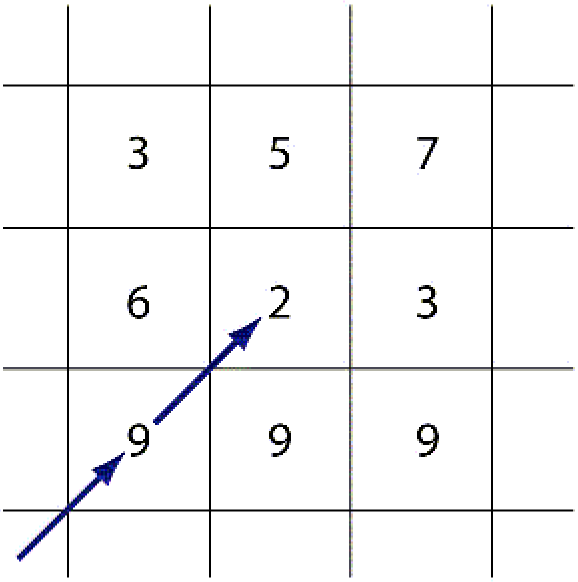
DTM data such as the data used in the example above actually contain quite a lot of uncertainties. The rigid grid structure doesn't account for geomorphologic features like ridges or canyons and there might be measurement errors, changes in the terrain surface that have not been updated or errors originating from preceding data processing procedures. Due to the nature of DTM uncertainties stochastic methods might yield a more realistic drainage net. The Rho8 algorithm is very similar to the D8 algorithm, but instead of square root of 2 it uses 2-*r*, where *r* is a uniformly distributed random number between 0 and 1, to estimate the diagonal distance between 2 adjacent grid points. This results in a different, eventually more realistic ldd net. See Moore in Burrough et al. (1998). Another alternative is to perform a Monte Carlo simulation by repeatedly randomising the elevation values in the DTM and extracting the ldd net. The most often occurring net can be taken as the most likely solution.

Multiple flow direction algorithms

One could argue that water might not only drain to a single cell but to several adjacent cells. The FD8 and the FRho8 algorithms address this problem by allowing dispersion. Moore (1996) recommends that a flow-dispersion algorithm (Quinn et al. 1991) be used to estimate specific catchment areas rather than the traditional D8 algorithm.

Removal of pits

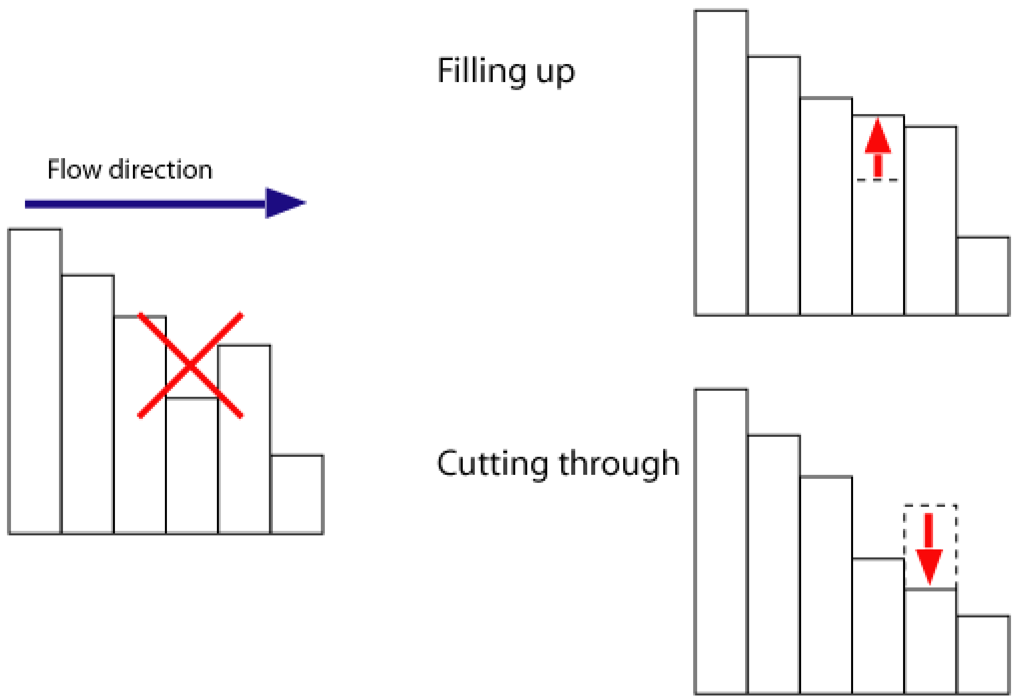
DTM's can have pits. Pits might represent the real geomorphology in some rare cases (glacial cirques, karst), but often they are DTM artefacts and need to be corrected in gridded DTM's. Pits frequently occur in narrow valleys where the width of the valley bottom is smaller than the cell size. A pit is a single cell that has a lower elevation than all surrounding cells, e.g. in the figure below the central cell with elevation=2. Therefore no water can drain and the drainage topology is disrupted.



The numbers in the above figures indicate the elevations of each raster cell. The central cell is a pit because the water can't drain anywhere.

Burrough (1998) suggests two strategies for removing pits (next figure):

- Cutting through:The search window around the pit cell is gradually increased until a cell or series of cells of the same elevation or lower than the pit cell is found. There might be more than one row of boundary cells that have to be cut through.
- Filling up: The elevation of the core pit cell is increased until the water can drain to one of its neighbours.



Filling up or cutting through as strategies to generate a continuous drainage net.

In reality the DTM doesn't necessarily have to be changed. The important aspect is that the drainage network becomes connected. Therefore, instead of filling up a pit or cutting through to the next cell, a pit could just be linked to the consecutive cell topologically without modifying the underlying elevation values.

Flat spots

Not only pits but also flat spots can cause problems in DTM's as the drainage direction is undefined here. Flat spots may be treated like pits, i.e. you could take the next lower raster cell as an outlet. Alternatively they might be seen as lakes, i.e. from any point in the flat region the water will drain to the entire flat area.

Fill up pits and find the drain directions in raster DEM

Below is a very small raster DEM with elevation values for each cell

- There is one pit. Identify it and fill it up by clicking.
- Where is the water from the source entering the lake? Click on the according cell inside the lake.

Only pictures can be viewed in this version! For Flash, animations, movies etc. see online version. Only screenshots of animations will be displayed. [link]

1.1.4. Information derived from a drainage network

An ldd net itself isn’t that useful yet. But given the fact you knew for each raster cell where the water drains to, wouldn’t it be possible to go the opposite way and count the number of upstream elements for a certain cell? And once you knew the upstream elements for a certain cell, wouldn’t it be interesting to find out how much water accumulates from all those cells?
An ldd net that has been derived it can be used for a large variety of specific hydrology related analysis. An obvious application is calculating upstream elements for computing the catchment areas, flow accumulation, ridges or stream channels. Scientists have proposed many hydrologic indices. Often they aren’t too difficult to calculate with ordinary map algebra software. The ones discussed here are:

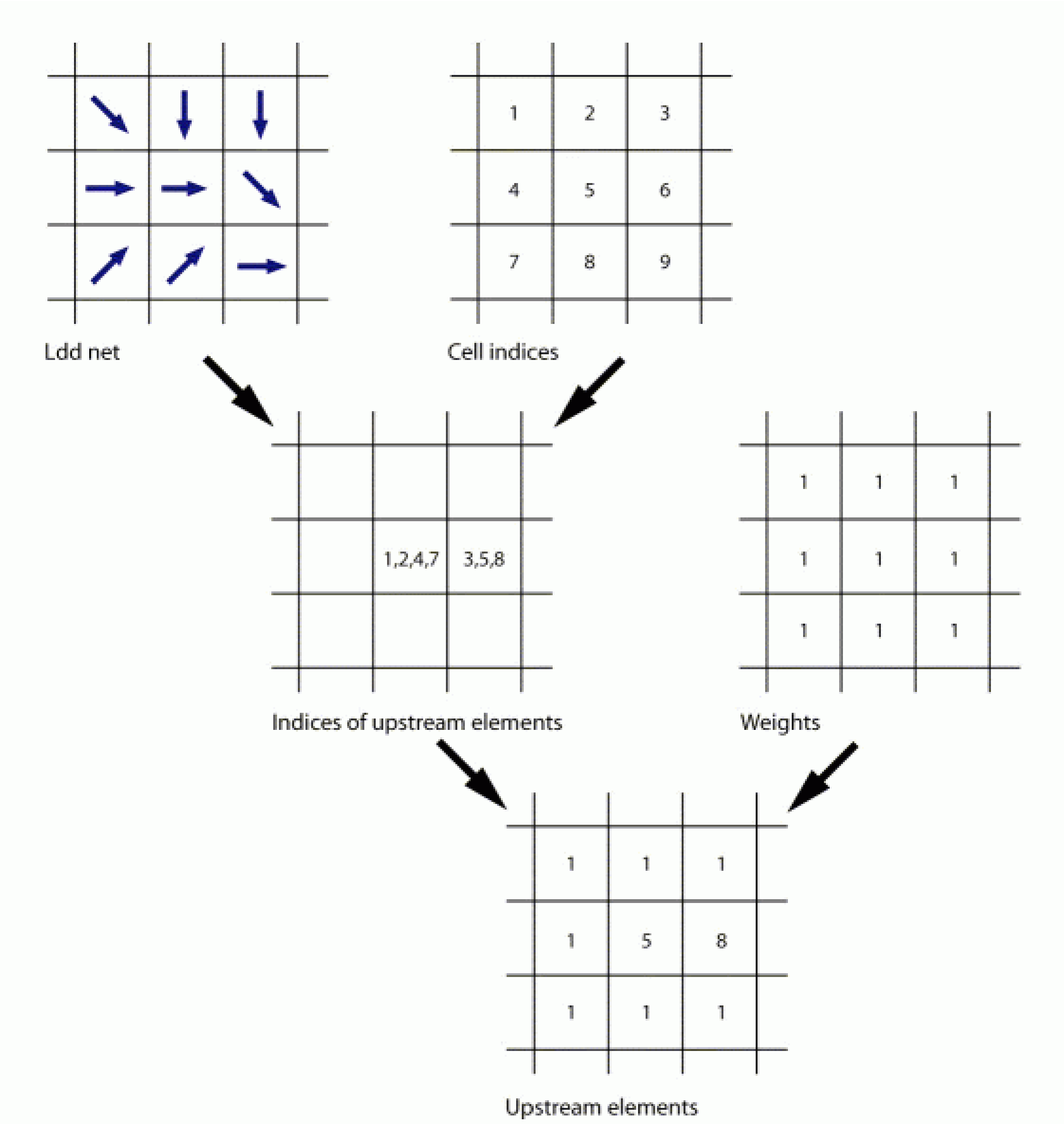
- Wetness index (CTI)
- Stream power index (SPI)
- Sediment transport index (STI)

Upstream elements (Flow Accumulation)
The upstream area is an important factor for the estimation of material fluxes. Given the ldd net Equation 2 makes it quite easy to calculate the number of upstream elements for each raster cell.

$$S(c_i) = S(c_i) + \sum_{u}^{n} (c_u)$$

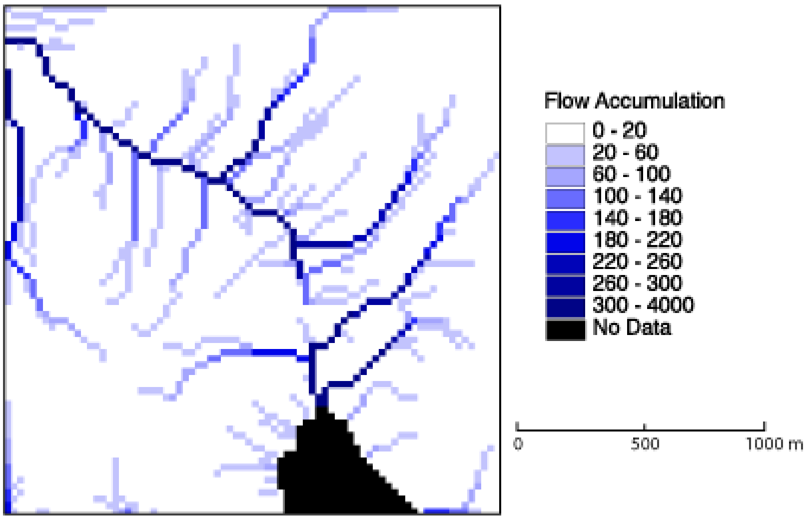
Upstream elements draining to a cell.

Where *ci* is the ith raster cell with value *S(ci)* and where *SUM(cu)* is the sum of all upstream elements draining to *ci*. Of course these upstream elements can additionally be weighted depending on the actual application. For a flow accumulation prediction the weights may be set according to the local amount of rain and the share of evaporated, infiltrated and intercepted water.
In the next illustration top left shows again the ldd net from the figure above. We could then provide an index for each cell as suggested in the illustration top right and find the upstream elements for each cell (middle left). Applying the equation shown above with the weights (middle right) results in a upstream elements matrix as shown at the bottom in the illustration.



Calculating the number of upstream elements for each cell.

In the illustration below the upstream elements were calculated for a realistic DEM. The resulting pattern looks quite similar as the river network on an according topographic map.



Flow accumulations can be computed from an ldd net. (Swisstopo 1991)

Stream channels and ridges

By setting a threshold value, it is possible to extract stream channels from a flow accumulation map. Unlike a flow accumulation map, a map showing stream channels or ridges merely contains boolean values, i.e. a cell either belongs to a stream channel/ridge or it doesn't. You will eventually have to adjust the threshold manually until you get a realistic result. Calculating stream channels by setting a threshold:

if(upstreamelements >= 50*) then streams*=true;

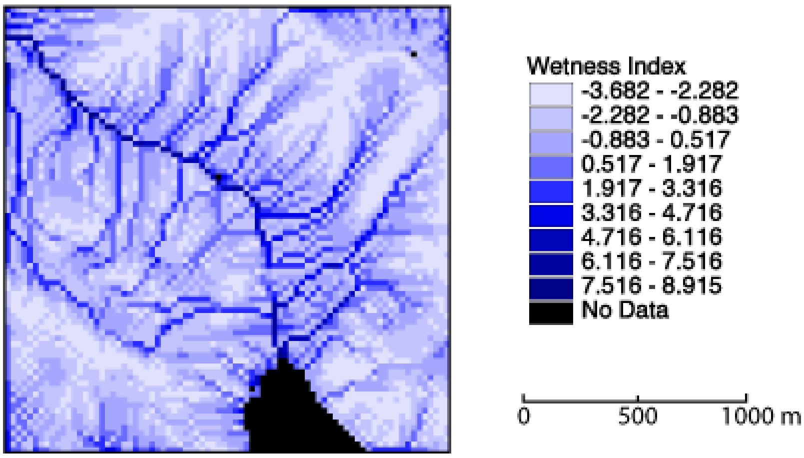
else streams=false*;*

Computing ridges in a raster can be seen as the dual problem of computing channels. Ridges are by definition cells with no upstream elements. Brändli (1997) developed and evaluated various methods for the extraction of geomorphologic and hydrologic features from DTM's.

Wetness index

The upstream element map can itself be useful for computing other hydrologic indices. A good example is the wetness index (see Beven and Kirkby (1979) in Burrough et al. (1998)) in the next equation, where *As* is the upstream area (number of upstream elements multiplied by the area of each grid cell) and is the slope at a given cell. The illustration below shows an example of a wetness index map. Obviously the highest values are in channels, the lowest on ridges.

$$wetness = \ln(A_s / \tan \beta)$$

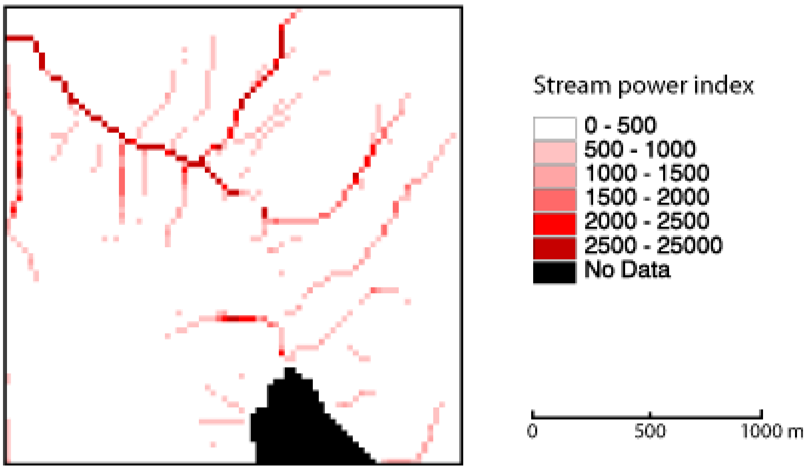


Wetness index derived from an ldd net. (Swisstopo 1991)

Stream power index

The stream power index (see Moore et al., 1993 in Burrough et al. (1998)) shown in the following equation is a measure for the erosive power of overland flow. *As* is the upstream area and is the slope in a given cell. In the following image, you can see that the stream power index map looks quite similar as the map of upstream elements.

$$stream\_power = A_s * \tan\beta$$

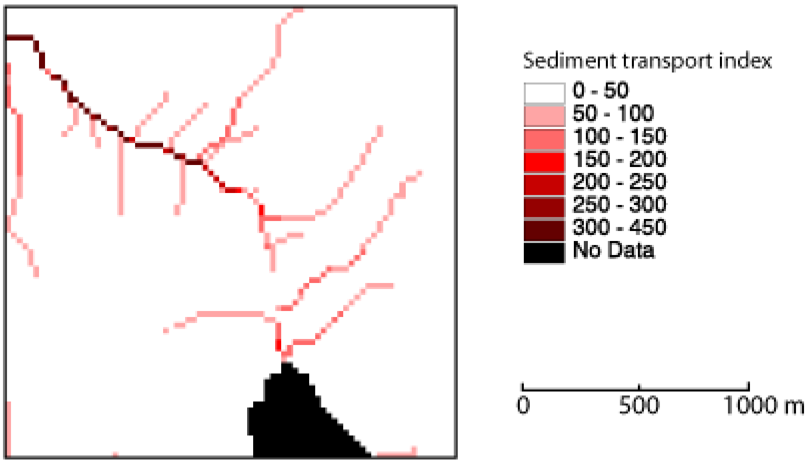


Stream power index derived from an ldd net. (Swisstopo 1991)

Sediment transport index

The sediment transport index characterises the process of erosion and deposition. Unlike the length-slope factor in the Universal Soil Loss Equation (USLE) it is applicable to three-dimensional surfaces (Burrough et al. 1998). The sediment transport index is defined by the equation below. *As* is the upstream area and is the slope at a given cell. The next illustration shows an example of a sediment transport index map. The upstream area is weighted stronger than the slope. Therefore the result is not the same.

$$STI = (A_s / 22.13)^{0.6} * (\sin\beta / 0.0896)^{1.3}$$

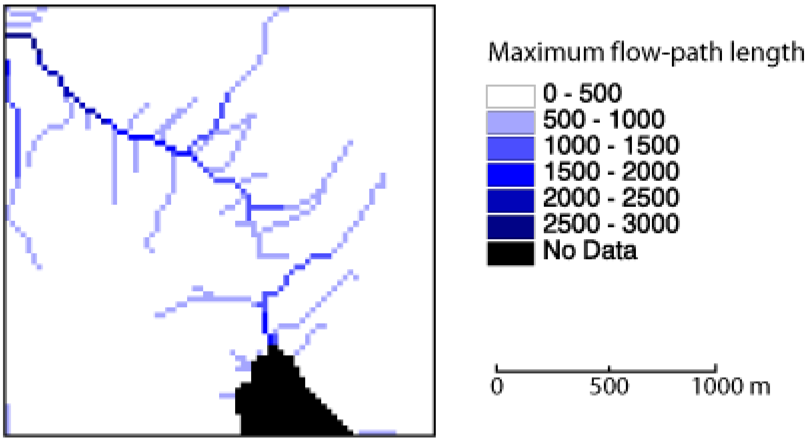


Sediment transport index derived from an ldd net. (Swisstopo 1991)

Maximum flow-path length

The maximum flow-path length is the length of the longest flow path from the catchment boundary to a given point in the DEM. Instead of accumulating areas like it is done for upstream areas, it accumulates flow distances across cells, and only the largest flow-path length of all upslope cells is passed on to the down slope cell, instead of the sum (Wilson et al. 2000).

The following illustration displays the maximum flow-path length for a real world example. Obviously hills and ridges have low values, channels have high values continuously increasing downstream.

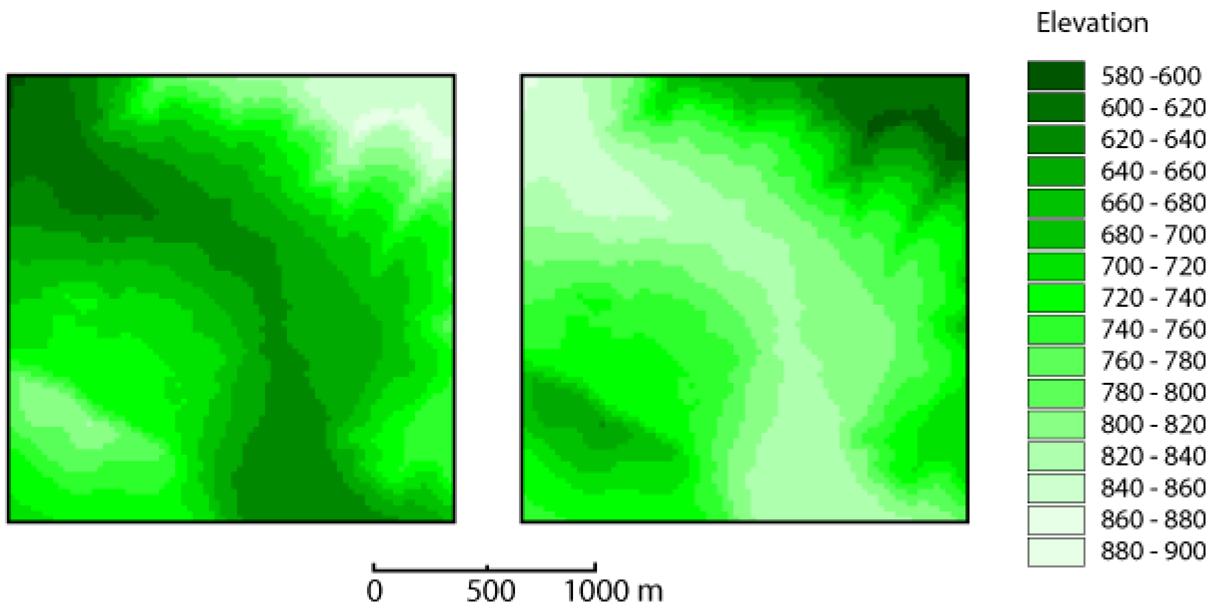


Maximum flow-path length in meters to each cell measured upstream. (Swisstopo 1991)

Down slope attributes

The indices described above are all based on the number of upstream elements, which we know how to compute. However, the opposite problem, i.e. you are asking for downstream elements, is also conceivable. Just imagine you wanted to map proximity to a stream. How would you find out something like that without adjusting the existing algorithm?

Basically the DEM has to be inverted (next figure) so that peaks become basins and vice versa. This can easily be achieved by subtracting all DEM values from a value higher than the highest peak in the original DEM. Using the algorithm for extracting upstream elements, you will get downstream elements which you can use for down slope analyses.



The original DEM (left) and its inversion (right). (Swisstopo 1991)

1.1.5. Questions

- What is an ldd net? Explain using your own words and post your answer on the discussion board. (Ask your tutors how they want to organize the discussion board).
- What steps (i.e. intermediary information products) are needed to compute a sediment transport index from a DTM?
- Why do TIN's have a better potential for hydrologic modelling than grids? Why are grids more favoured nevertheless?
- You want to compute downstream proximity to a flow channel but your GIS does not support that procedure. You have the idea to invert your DTM and use an upstream function. Which one would you choose?

1.1.6. Multiple choice quiz

1. An ldd net contains:

1. The flow direction at each cell in a grid
2. The amount of water draining into each grid cell
3. Streams and rivers

Answer to question 1

Answer 1 is correct!

2. The number of upstream elements of a certain grid cell in a DTM depends

1. Only on the underlying DTM
2. Only on the derived ldd net

3. On the DTM and the derived ldd net
4. On the DTM, the derived ldd net, and various other factors

Answer to question 2

Answer 2 is correct!

3. What is the advantage of multiple flow direction algorithms?

1. Unlike the D8-algorithm they can be used more than once
2. They are based on random numbers
3. They allow water drainage to more than one adjacent cell
4. They are particularly fast

Answer to question 3

Answer 3 is correct!

4. What is the problem with flat spots and pits in DTM's when performing hydrologic analyses?

1. The water drains to the wrong direction
2. The net is interrupted because the water can't discharge from there
3. Flat spots and pits don't occur in nature

Answer to question 4

Answer 2 is correct!

5. How many flow directions can be modelled in grids?

1. 4
2. 8
3. 9
4. infinite

Answer to question 5

Answer 2 is correct!

Choose the correct derived product in the flash animation below:

Only pictures can be viewed in this version! For Flash, animations, movies etc. see online version. Only screenshots of animations will be displayed. [link]

1.1.7. Summary

Gridded DTM's can be used for a large variety of hydrologic modelling. In a first step, pits have to be removed by filling up or cutting through and flat spots have to be dealt with as the drain direction is undefined here. Then a local drain direction net (ldd net) can be extracted from the DTM using either a single flow algorithm or an algorithm that allows dispersion. The actual choice of one algorithm depends on the terrain data and the nature of the application. The ldd net can be used to calculate parameters like the upstream area of a grid cell or the length of a catchment. Some geomorphologic features like ridges or flow channels are closely related and can also easily be derived. These parameters may be used in hydrologic indices or they may be combined with other geographic information in order to make runoff predictions or to model occurrences of plants and animals.

1.2. Visibility analysis and related topics

There is a live web cam on Saentis. Is Lake Constance visible from there right now? If it is not visible is it because of the terrain? See the **Säntis Webcam**.
The question of whether something is visible from a viewpoint is asked quite often and in different variations. Visibility of an object in the terrain depends on quite a lot of factors like terrain, atmospheric conditions, and soil coverage. We will focus on terrain here, but keep the other factors in mind.
The table below lists some questions that are related to visibility. They are all based on the same basic visibility problem but some of them require further steps in order to be answered. The column on the right assigns terms that will be discussed later in this unit.

Question	Related terms
Can a place in the terrain be seen from a certain viewpoint?	Intervisibility, line of sight (LOS), profile
What area is covered by the services of a radio antenna?	Viewshed
How can an aesthetically unpleasant building be optimally hidden in the terrain in order to preserve the beauty of a landscape?	Viewshed (the other way round)
Where is the horizon to a given viewpoint?	Horizon line
Where can radio antennas be placed in order to minimise costs and optimise services?	Viewshed, watchtower problem
What is the potential direct solar irradiance at a certain location?	Irradiance mapping, cast shadows

Learning Objectives

- You know what external effects have to be taken into account when using terrain data for a visibility analysis.
- You understand how a line-of-sight (LOS) check can be performed for grids.
- You know how questions related to the watchtower problem and moving viewpoints differ from simple intervisibility problems.
- When you perform a visibility analysis, you know where uncertainties arise from and you know ways to deal with them.
- You know the input parameters for modelling the potential direct solar radiation in rugged terrain.

1.2.1. External effects

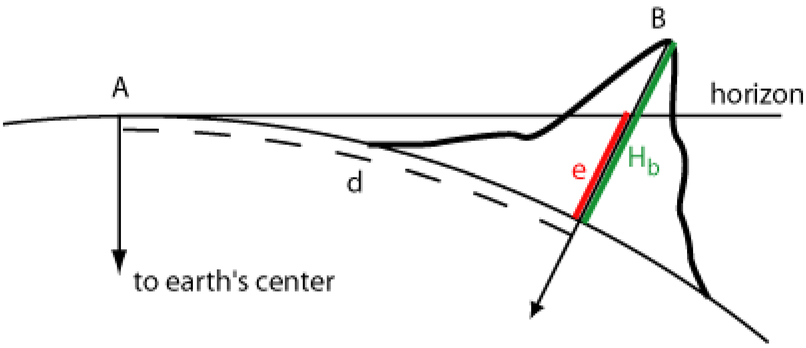
They all laughed at Christopher Columbus when he said the world was round…
(Frank Sinatra, Lyrics by Gershwin/Gershwin)

Zernez near the Swiss National Park photographed with a fisheye lens. (Photo Roland Schmidt)

The earth's curvature is not as pronounced as the picture above suggests. It was made with a special camera lens. Nevertheless prior to calculating the actual visibility, two important external effects may need to be compensated for (on occasion):

- Distortions of the DTM surface due to the earth's curvature and atmospheric refraction.
- The effects of land cover and other obstacles.

Earth's curvature



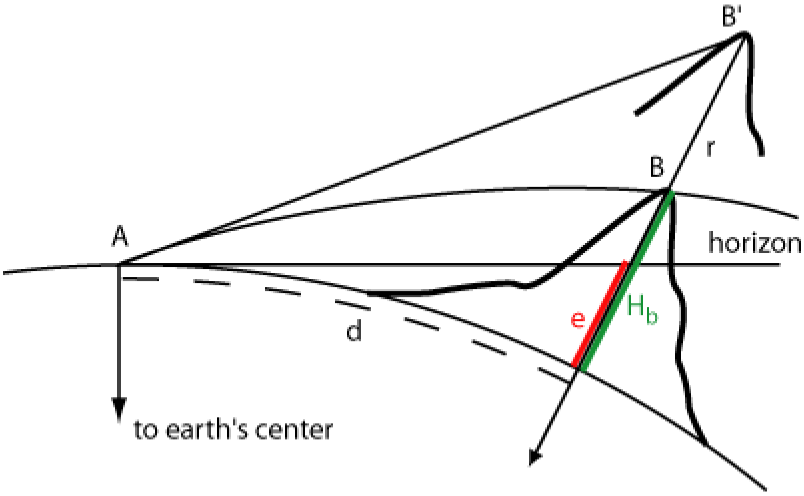
Effect of the earth's curvature.

The effect due to the earth's curvature is well known from ships disappearing behind the horizon. It is illustrated schematically in the figure above. Because of the earth's curvature, points are observed at lower elevations. In the figure, point *B* which is seen from viewpoint *A* and has height *H*, has disappeared beneath the horizon by the difference *e*. This effect can be approximated as follows (Yoeli 1985):

$$e = d^2 / 2R$$

where *d* is the distance between the viewpoint and the observed point and *R* is the radius of the earth (*R* = 6370 km). Of course the effect increases with growing distance *d*.

Atmospheric refraction



Atmospheric refraction.

Atmospheric refraction makes objects appear at higher angles than would be expected. This effect is caused by the fact that rays of light passing through the atmosphere are refracted or bent from a straight path in a direction towards the earth's surface under normal conditions of pressure gradient and temperature. Thus, as shown in the figure, point *B* is observed at *B'*, with the height difference *r*. This effect can be approximated as follows (Yoeli 1985):

$$\Delta h = d^2 \cdot k / 2R$$

where *k* is an empirical coefficient (*k* = 0.13) and *R* is the radius of the earth (*R* = 6370 km). Again the effect increases with growing distance *d*.

Correcting the effects of the earth's curvature and atmospheric refraction

The combination of the opposing effects of the earth's curvature and of atmospheric refraction leads to the correction formula

$$\Delta h = 0.87 \cdot d^2 / 2R$$

Where *d* is the distance between viewpoint and observed object and *R* is the radius of the earth (*R*=6370 km). This difference, of course, is negligible for short viewing distances. However, for longer distances it becomes quite considerable. Therefore all heights of the DTM should first be corrected depending on their distances to the viewpoint, yielding a corrected DTM' with heights . In the table below the equation above was used to calculate the combined effect of the earth's radius and atmospheric refraction for various distances *d*. The earth's radius is assumed 6'370'000 m. Thus for a distance of 5 km the effect is 1.71 m, whereas for a distance of 50 km it is 170.72 m.

Some examples for the combined effect of the earth's curvature and the atmospheric refraction for various distances *d*:

d[m]	5000	10000	20000	30000	40000	50000
	1.71	6.83	27.32	61.46	109.26	170.72

At what distance does the combined effect of the earth's curvature and the atmospheric refraction become bigger than 1 m?

3827 m

Other atmospheric interferences

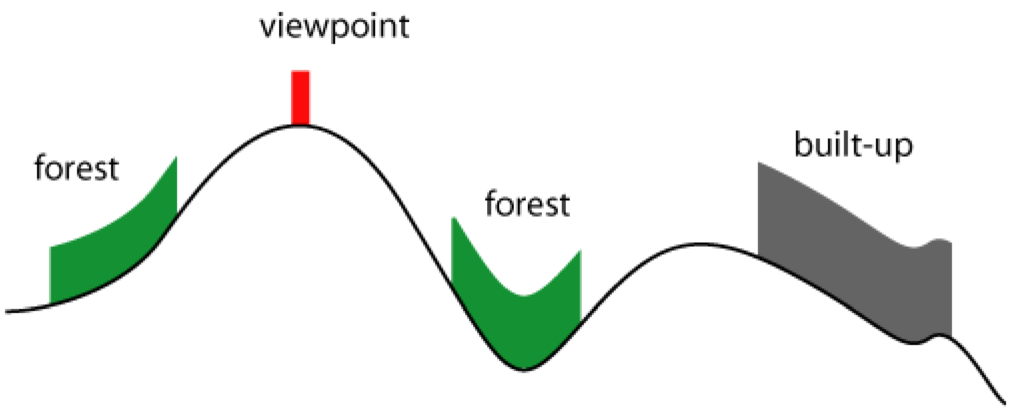
Effects such as haze, fog, clouds, or smoke, obviously can have a great influence on the visibility of terrain. Reasons why these effects are often ignored is that they are not trivial to model, and also subject to rapid change. For some applications like radio signal transmission, atmospheric haze and fog do not matter that much as radio waves can pass through.

Various visualization software tools allow the modelling of atmospheric influences. Using this software the visual appearance of a landscape may be simulated if you have suitable terrain data at your disposal. In the next figure the landscape around Niesen and Bernese Oberland is modelled under various whether conditions and with different plant coverages. See how haze darkens the mountains in the background.

Niesen and the Alps of the Bernese Oberland under different atmospheric conditions at different times of the year. (Swisstopo 1991)

Obstacles and land cover

Another factor that influences the visibility of terrain is the land cover. Certain land use classes, such as forest or built-up areas, can form visual obstacles of large spatial extent and considerable height. For a more realistic solution of the visibility problem, the heights of relevant land cover types – as well as other potential natural and man-made obstacles – have to be taken into account. If digital land cover data are available, an estimated average object height can be added to each DTM point depending on which land cover types they fall in. Certain photogrammetric methods tend to capture the top of trees and buildings rather than the ground surface. Obviously, this presents a certain advantage in the use for visibility applications.



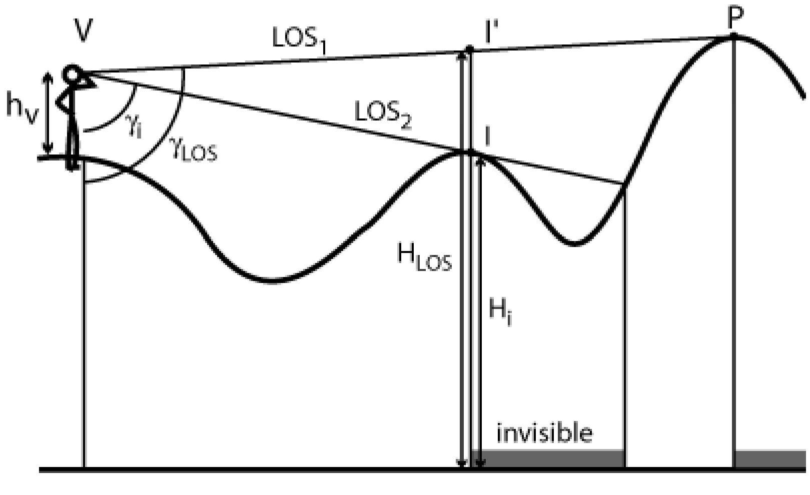
Effect of land covers on visibility analyses.

1.2.2. Line-of-sight problem

The basic principle of visibility computation is based on the line-of-sight (LOS) problem. The problem of determining the LOS between a viewpoint *V* and an observed point *P* is illustrated in the next figure. Note that the viewpoint is raised by a height *hV*, which relates to the height of an observer or a watchtower. The profile between *V* and *P* needs to be tested for obstructions, which might break the LOS.

The LOS is broken:

- if either any point between *V* and *P* has a vertical viewing angle which is larger than the vertical angle of the LOS.
- or if the height *HLOS* of the LOS at any point is lower than the elevation *Hi* of that point.

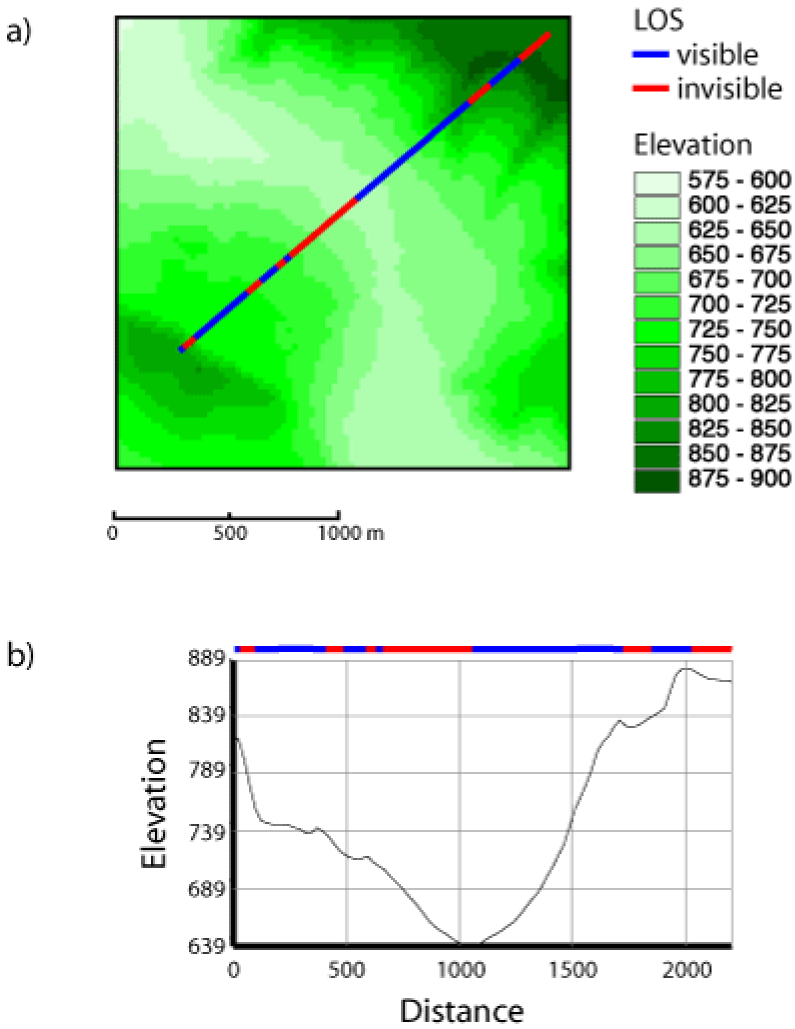


The line-of-sight (LOS) problem.

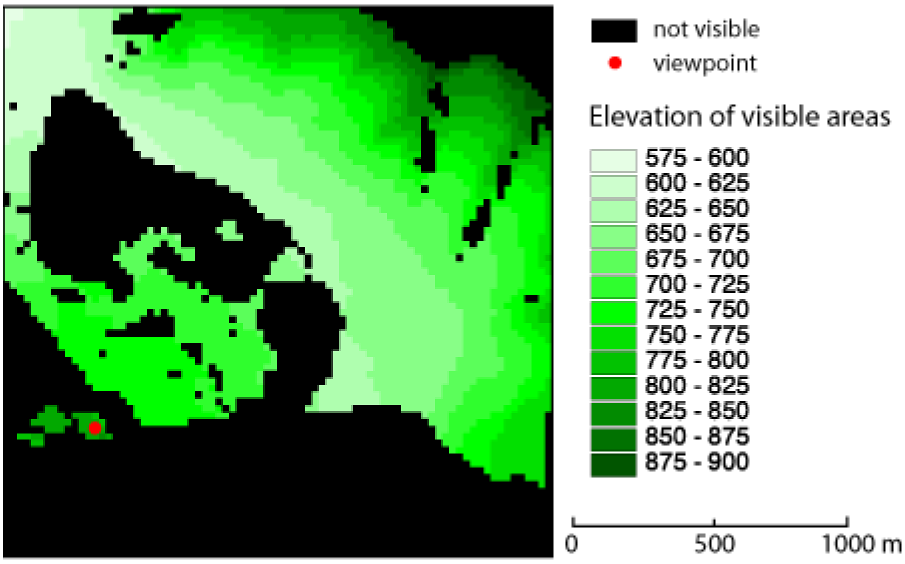
Algorithms to solve the LOS problem (LOS checks) depend on the data structure used to represent the DTM. While the basic principle of visibility is valid in any case, an implicit spatial order, as it is inherent for gridded DTM's, greatly simplifies the determination of visibility.

Line-of-sight check for grids

A line-of-sight check requires a terrain profile between a viewpoint and a target point. An LOS check is performed for each new vertex inferred. The computation can be terminated as soon as an LOS blockage is found or the target point is reached. For a viewshed analysis the LOS between the viewpoint and each other point has to be tested and is therefore quite demanding. There are examples for a line-of-sight and an entire viewshed in the next two figures.



Line-of-sight through a grid. a) View from above. b) Profile view. Red sections are invisible, blue sections are visible from the viewpoint in the southwest. (Swisstopo 1991)



Viewshed from one observer point in the center of the map. (Swisstopo 1991)

Line-of-sight check for TIN's

The solution of the visibility problem for TIN's is substantially more complex than for grids because of two facts:

- An implicit spatial order is not established in TIN's. Thus, any LOS calculation involves a fair amount of TIN traversal.
- In contrast to grids, TIN's are made up of irregular facets of different size and shape. While in the case of grids, it is considered sufficient to sample the visibility by checking the visibility of individual grid cells, each TIN facet needs to be checked whether it is fully, partially, or not at all visible from the viewpoint in order to obtain a complete solution.

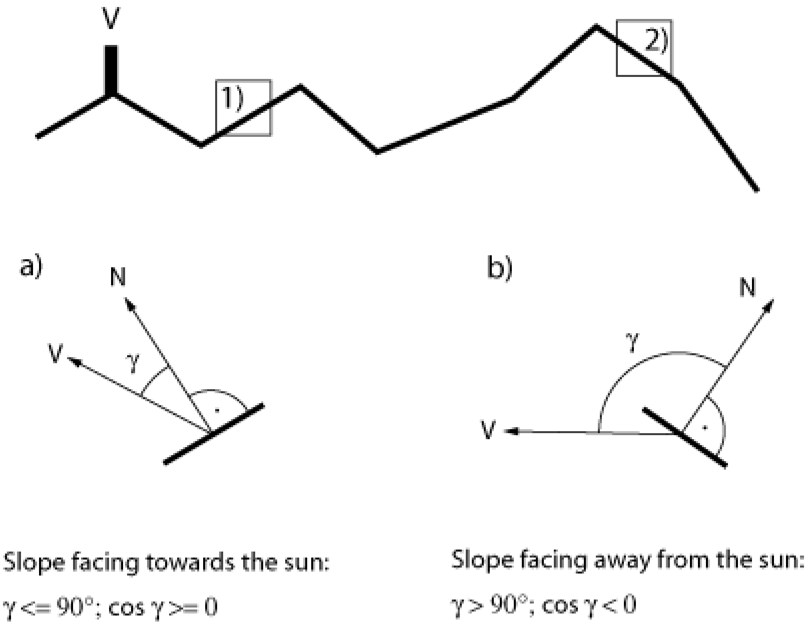
In principle, any so-called object-space hidden surface algorithm, see Sutherland et al. (1974) and Foley et al. (1992), which is used to determine the visible parts of arbitrary 3-D objects, could be applied to extract visibility regions on a TIN. However, as TIN's represent more specific structures (they are 2.5-D, and facets are restricted to triangles), simpler algorithms can be designed based on heuristics. For instance, backface culling (see below) significantly reduces the number of triangles that have to be tested for visibility, which is quite an effort. Such heuristics are particularly useful if the number of visibility analyses required is very big, e.g. if moving viewpoints occur.

Backface culling

As visibility computations in TIN's are very demanding, clever algorithms try to somehow reduce the amount of work to be done. Backface culling accelerates visibility analyses by making use of the following logical implication:

- Any triangle facing away from the viewer can immediately be rejected as invisible.

Triangles facing away can be found by calculating the normal vector for each TIN facet on the profile along the line-of-sight (figure below). If the angle between the normal and the vector to the viewpoint is larger than 90° it is certain that the facet is invisible (figure below b)).



Triangles facing away from the sun (b)) don't have to be tested for visibility.

Depending on the position of the viewpoint and on the terrain type, it is not uncommon that the number of triangles, which must further be checked (figure above a)) is reduced by 50 per cent. An algorithm based on such heuristics has been presented by De Floriani et al. (1986).

1.2.3. Moving objects

In the sections above describing the line-of-sight problem, the viewpoint was fixed. If the viewpoint moves around, its intervisibility with a target point in the terrain has to be recomputed for every time step. In the case of an entire viewshed, this requires a lot of processing power and/or more specific algorithms and sophisticated heuristics like backface culling. Increased processing power can be achieved by parallel processing, which is particularly suitable for grids because of their regular topology. You can find out more about high performance computing and GIS in Longley et al. (1999).

1.2.4. Watchtower problem

The visibility problem can basically take two forms:

1. For single or multiple viewpoints, find the viewshed within a given study area.
2. Find the minimum set of viewpoints, so that any part of the entire study area can be seen from at least one viewpoint.

The first problem is the classical visibility problem (What can I see from a given location?). It can be reduced to a number of LOS checks. The second task could be termed the "watchtower problem" (Where do I place my watchtowers so I can see the entire surface). It is obviously more complex to solve, since the number of locations that have to be tested is not known in advance. A solution for connecting a given set of transceiver stations with a minimum set of relay stations, a problem related to the watchtower problem, can be found in De Floriani et al. (1992).

Inevitably DTM's are uncertain due to unknown errors in the data and in the modelling assumptions. These uncertainties propagate through any calculations performed and can have significant effects on the quality of the result. For a visibility analysis this means that in reality a certain object in the landscape might (not) be visible from a given viewpoint although a GIS-based visibility analysis undoubtedly gives the opposite result. Usually visibility maps conceal that they are created by simply giving a boolean result of either visible or not visible areas. Adequate visualisation can support the user reading the visibility map; however, it does still not provide a quantitative expression of the viewsheds uncertainty. Monte Carlo simulations yielding fuzzy viewsheds may be suitable for dealing with uncertainties in visibility analyses.

Monte Carlo simulation of uncertainties

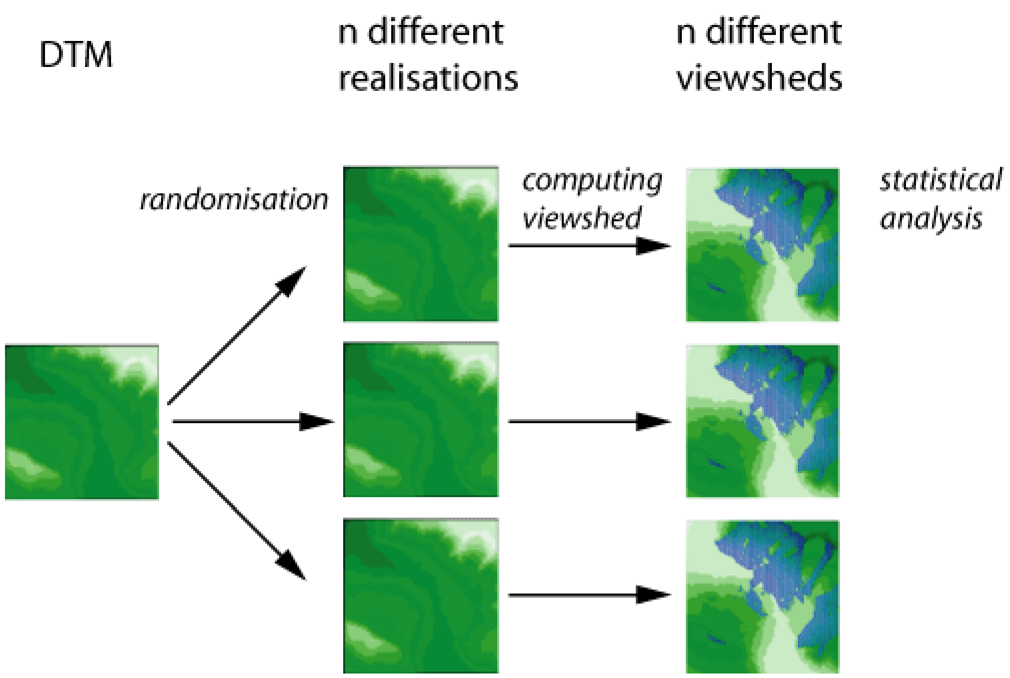
Monte Carlo methods are those in which properties of the distributions of random variables are investigated by use of simulated random numbers (Kotz et al. 1985).

An excellent example of a Monte Carlo Simulation is the empirical estimation of the number with Buffon's needle experiment. Buffon repeatedly dropped a needle on a tabletop with a number of parallel lines on it. He estimated by counting the number of times the needle intersected or touched a line and by the number of times it didn't. To be honest it is quite a pain to do it like Buffon once suggested back in the 18th century. Nowadays you can settle back and let a **Java Applet** do Buffon's needle experiment.

In digital terrain modelling errors in the DTM's are supposed to be randomly distributed. Monte Carlo methods therefore try to realistically simulate the spatially correlated (i.e. autocorrelated) metric elevation errors in the data points.

Using a pseudo random number generator n different realisations of the DTM error as expressed by the RMSE are generated and added to the heights of the original DTM, yielding n differently perturbed DTM's. Each of these DTM's will result in a different viewshed. They are then combined to yield a map, which represents the viewshed in terms of "number of times seen" instead of a binary classification into visible/invisible. Thus, certainty factors for the visibility of each DTM points can be derived, yielding a "fuzzy viewshed" (Fisher 1991).

The next figure schematically again depicts how a Monte Carlo simulation in the case of a viewshed analysis works. Randomising the terrain data yields *n* different realisations of the DTM and therefore probably n different viewsheds. The data can finally be analysed statistically in order to receive information about the uncertainties in the output.

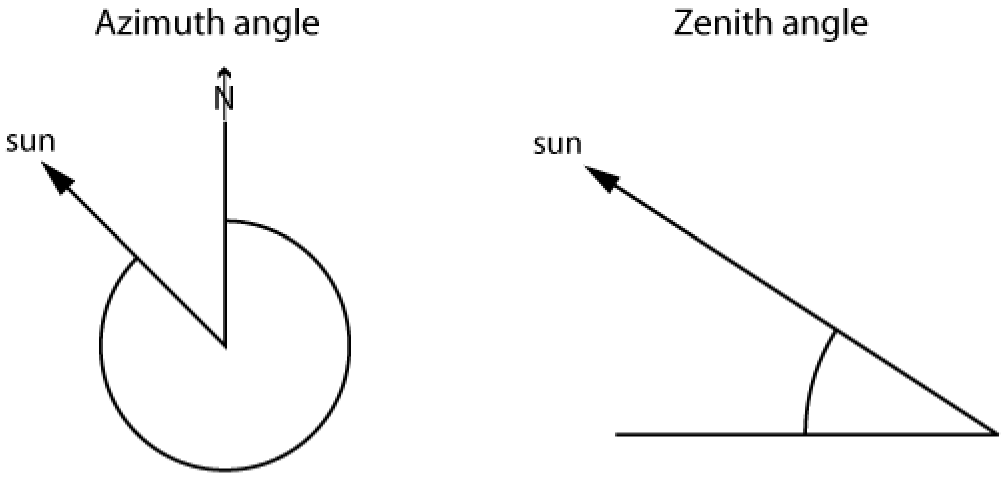


Schema of a Monte Carlo simulation for a viewshed analysis.

As Fisher (1991) notes, when modelling DTM elevation errors it is important to take spatial autocorrelation between the error values into account. He suggests an algorithm based on a method outlined by Goodchild (1980) for simulating autocorrelation. Other more sophisticated methods can be found in Chilès and Delfiner (1999).

1.2.5. Maps of topographic shadows

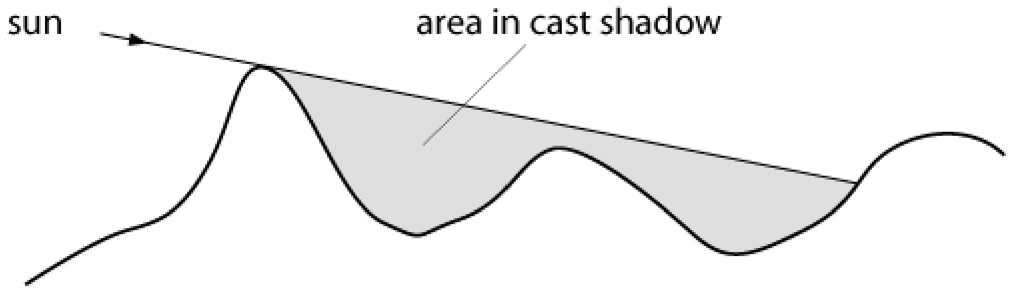
The determination of topographic shadows is closely related to visibility problems. Just imagine that the viewpoint is located on the sun. Topographic shadow areas are all places without direct solar radiation.
Because the sunrays can be considered parallel, instead of the viewpoint coordinates the suns direction and zenith angle are used. Direction and zenith angle are independent on the location in the DTM. Don't confuse maps of topographic shadows and shaded relief maps or hillshading.



Zenith and azimuth angle for indicating the position of the sun.

Topographic shadows can occur due to two effects:

- Some slopes of the terrain surface will face away from the incident sunrays and thus be in the shade. This effect is purely local, and the context does not matter. You might use backface culling to extract these areas very quickly.
- Shadows can also be cast by the terrain nearby (next figure). This case is more complex since the topographic context must be taken into account.

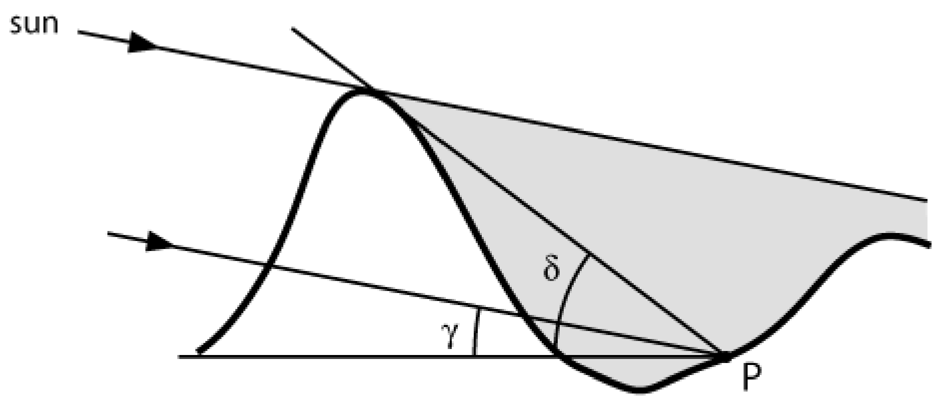


Shadows can be cast by terrain nearby. Therefore areas may lie in the shade even though they actually face the sun.

If a slope faces the sun the question is, whether the sun lies above or below the horizon. The situation is depicted in the figure below. Thus, a point is in cast shadow if the zenith angle # to the horizon in the direction of the sun is bigger than the zenith angle of the sun. The zenith angle of the horizon can be calculated using the following formula:

$$\delta = \arctan\left(\frac{\Delta z}{d}\right)$$

where is the altitude difference between the horizon and the target point *P* and *d* is their distance. If is smaller than , *P* lies in the sun, else *P* lies in the shade. Of course zenith angle and direction (azimuth angle) of the sun must be given.



A point P lies in the shade if its zenith angle to the sun (gamma) is smaller than its zenith angle to the horizon (delta).

As you've seen, once the horizon at a certain point is known it is relatively straightforward to find out whether a point is in the shade or in the sun. In the case of the analysis of potential direct solar irradiance within a given time period, the position of the sun changes continually. Therefore, cast shadows have to be computed many times for different positions of the sun. Thus, it might be a good idea to extract and store the horizon lines to each grid point in advance and use them in each shadow computation again and again.

1.2.6. Horizon lines

A point belongs to the horizon of a given viewpoint *V* in the terrain if there is no other point farther away in the same direction that is still visible from *V*. Algorithms extracting horizon lines are quite complicated and they aren't explained in detail here. One of them (for grids) can be outlined with the following steps:

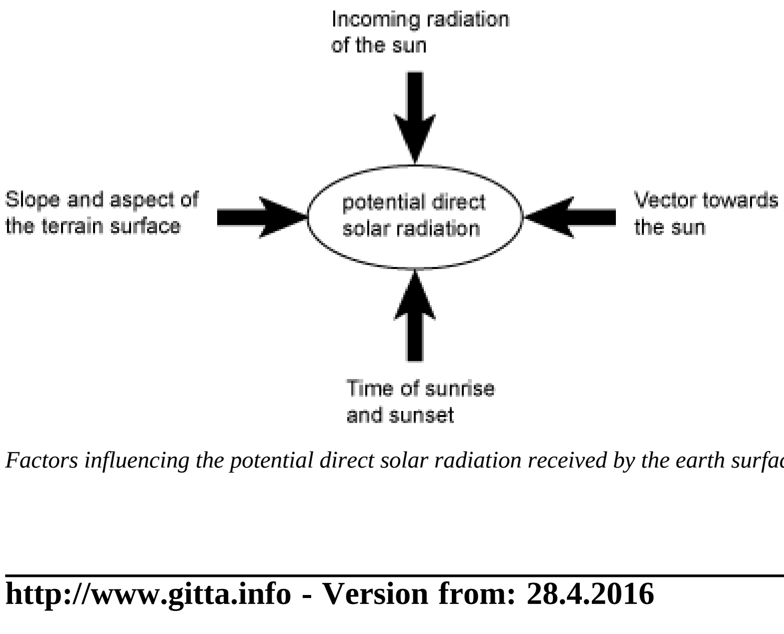
1. Derive a terrain profile from north to south through a point *V* of the grid.
2. Compute *V*'s horizon points in forward and backward direction i.e. north- and southwards
3. Rotate the grid by a small angle.
4. Repeat steps 1 to 3 until the grid is rotated by 180° and sufficient horizon points are known.
5. Repeat steps 1 to 4 for all grid points.

Thus the result of the algorithm is a horizon line for every grid point. More details about the algorithm as well as a pseudo code version are given in Dozier et al. (1981).

1.2.7. Potential direct solar radiation

Potential direct solar radiation
Earlier in the unit we already mentioned some applications of visibility analyses like viewsheds, the watchtower problem, or maps of topographic shadows. In this section, we are setting sail for a more advanced topic that incorporates many of the terrain analysis operations discussed above. You will see that, with what you know about DTMs, you are ready to accomplish the complicated task of modelling potential direct solar radiation. Good resources for additional information on solar radiation modelling are Funk et al. (1992) and Corripio (2003).

Short problem description
The potential direct solar radiation is the energy of the solar radiation received by a particular surface point in one day under clear sky conditions. It depends on the components depicted in the figure below.



Factors influencing the potential direct solar radiation received by the earth surface.

The potential direct solar radiation can be expressed mathematically as:

$$I_P = \int_{t_r}^{t_s} I_0 * \cos(N,S)dt$$

where is the incoming radiation of the sun, slope and aspect of the terrain surface are expressed by the surface normal *N*, *S* is the vector towards the sun, and *tr* and *ts* are the time of sunrise and sunset (Funk et al. 1992).

Incoming solar radiation

For the determination of the incoming solar radiation , empirical models have been developed such as the one given in (Funk et al. 1992).

Surface normal

The surface normal *N* can be calculated as the vector product of the surface derivatives in x- and y-direction.

Vector towards the sun

The vector towards the sun *S* depends on latitude of the geographic position, the day of the year (i.e. the declination of the equatorial plane), and the time of the day. If all these parameters are given *S* can be calculated and indicated by means of two angles, the zenith angle and the azimuth angle (Funk et al. 1992).

Time of sunrise and sunset

Finally, the time of sunrise *tr* and sunset *ts*, are defined by the condition that the vector towards the sun *S* and the vector towards the horizon are identical. After determining *tr* and *ts* for each grid point and each azimuth angle of the sun the number of hours each grid point is in shadow (or sun, respectively) can be mapped.
There's an **Sun, Moon and Earth applet** that allows to determine the position of the sun. Play around with it and see what you can find out. At what azimuth and zenith angle was the sun on September 16 2003 at 17:20 viewed from Zurich?

Zenith angle: 26.8°; Azimuth angle: 251.3°.

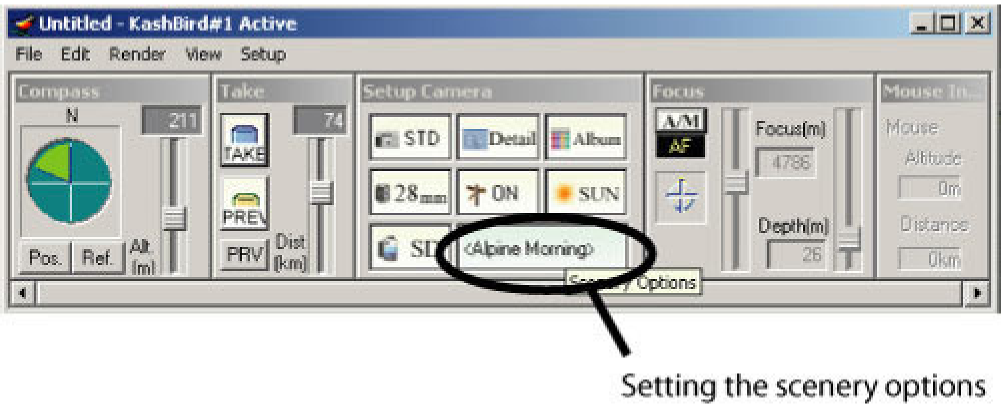
1.2.8. Exercise curvature and atmospheric refraction

Correcting the effects of the earth's curvature and atmospheric refraction

Question: The DHM25 from swisstopo (1991) in flat or hilly areas has an elevation accuracy of about 1.5 m. Is it worth correcting for earth curvature and atmospheric refraction if the maximum viewing distance is less than 5 km?
Discuss this question in your (online) community.

1.2.9. Exercise atmospheric interferences

Kashmir3D is a 3D visualisation software. You can download it for free from the **Kashmir 3D Website**. Try to visualise a terrain data set of your choice (e.g. the Matterhorn test data available from **Swisstopo**) using the KashBird module of Kashmir3D and play around with the scenery options.



Setting scenery options of the KashBird module from Kashmir3D.

1.2.10. Multiple choice quiz

1. The distance between Romanshorn and Friedrichshafen is 13 km. By how much is Friedrichshafen disappearing behind the horizon looking from Romanshorn across Lake Constance?

1. 11.54 m
2. 12.54 m
3. 13.54 m
4. 10.54 m

Answer 1 is correct!

2. Is there any direct solar radiation inside cast shadow areas?

1. No
2. Yes

Answer 1 is correct!

3. A TIN facet is invisible from a viewpoint if in the profile along the line-of-sight the angle between its normal vector and the vector towards the sun is:

1. >90°
2. <90°

Answer 1 is correct!

4. For a line-of-sight check you need to derive a profile first

1. True
2. False

Answer 1 is correct!

5. Backface culling can make a visibility map:

1. More precise
2. Faster to compute
3. More accurate
4. More reliable

Answer 2 is correct!

6. If we have to place several viewpoints in an optimal way, we call that:

1. Multiple viewpoint problem
2. Watchtower problem
3. Moving viewpoint problem
4. Parallel computing problem

Answer 2 is correct!

7. A Monte Carlo simulation is useful to investigate:

- Various viewsheds from a viewpoint
- Viewsheds from various viewpoints
- Uncertainty propagation to the output of a GIS analysis

Answer 3 is correct!

8. The time of sunrise and sunset depends on:

1. Geographic longitude
2. Weather conditions
3. Haze and fog
4. Terrain
5. Geographic latitude

Answers 1, 4 and 5 are correct!

1.2.11. Summary

Visibility analyses belong to the most popular applications of digital terrain models. They are based on the so-called line-of-sight problem (LOS problem) in which a profile is intersected with the straight connection between viewpoint and observed object in the terrain. Visibility analyses are subject to external effects, i.e. the earth’s curvature and atmospheric refraction. Those have to be taken into account and they possibly have to be corrected. More complex applications of the LOS problem are the viewshed analysis and the watchtower problem. It is also quite demanding to consider moving objects. Errors in the DTM propagate to the output

of a visibility analysis. Monte Carlo simulations are a way to deal with these uncertainties. The calculation of topographic shadows, horizon lines and the potential direct solar radiation are also variants of visibility analyses. The same concepts are valid here.

1.3. Bibliography

- **Brändli, M.**, 1997. *Modelle und Methoden für die Extraktion geomorphologischer und hydrologischer Objekte aus digitalen Geländemodellen*. . Geographisches Institut der Universität Zürich.
- **Burrough, P. A., McDonnell, R. A.**, 1998. *Principles of Geographical Information Systems*. New York: Oxford University Press.
- **Chilès, J., Delfiner, P.**, 1999. *Geostatistics: Modeling Spatial Uncertainty*. New York: John Wiley and Sons.
- **Corripio, J.**, 2003. Vectorial algebra algorithms for calculating terrain parameters from DEMs and solar radiation modeling in mountainous terrain. *International Journal of GIS*, 17(1), 1-23.
- **De Floriani, L., Falcidieno, B., Pienovi, C., Allen, D., Nagy, G.**, 1986. A Visibility-Based Model for Terrain Features. *In: Proceedings Second International Symposium on Spatial Data Handling*. Seattle, 235-50.
- **De Floriani, L., Nagy, G., Puppo, E.**, 1992. Computing a Line-Of-Sight Network on a Terrain Model. *In: Proceedings 5th International Symposium on Spatial Data Handling*. Charleston, 672-681.
- **Dozier, J., Bruno, J., Downcy, P.**, 1981. A Faster Solution to the Horizon Problem. *Computers & Geosciences*, 7, 145-51.
- **Fisher, P.F.**, 1991. First Experiments in Viewshed Uncertainty: The Accuracy of the Viewshed Area. *Photogrammetric Engineering and Remote Sensing*, 57(10), 1321-1327.
- **Foley, J.D., van Dam, A., Feiner, S.K., Hughes, J.F.**, 1992. *Computer Graphics: Principles and Practice*. Reading: Addison-Wesley. [Second Edition, Revised Fifth Printing.]
- **Funk, M., Hölzle, M.**, 1992. A Model of Potential Direct Solar Radiation for Investigating Occurrences of Mountain Permafrost. *Permafrost and Periglacial Processes*, 3, 139-142.
- **Goodchild, M.F.**, 1980. Algorithm 9: Simulation of Autocorrelation for Aggregate Data. *Environment and Planning*, 12, 1073-1081.
- **Gruber, S., Hölzle, M**, 2001. Statistical modeling of mountain permafrost distribution: Local calibration and incorporation of remotely sensed data. *Permafrost and Periglacial Processes*, 12(1), 69-77.
- **Kotz, S., Johnson, N. L.**, 1985. *Encyclopedia of Statistical Sciences*. New York: John Wiley and Sons.
- **Longley, Paul A., Goodchild, Michael F., Maguire, David J., Rhind, David W.**, 1999. *Geographical Information Systems. Principles, techniques, applications and management*. New York: John Wiley and Sons. [2 volumes. 580 pages. 2nd edition]
- **Moore, I. D.**, 1996. Hydrological Modeling and GIS. *In:* **M. F. Goodchild, L. T. Steyaert, B. O. Parks, C. Johnston, D. Maidment, M. Crane, and S. Glendinning**, ed. *GIS and Environmental Modeling: Progress and Research Issues*. Fort Collins, Colorado: GIS World Books.
- **Quinn, P., Beven, K., Chevalier, P., Planchon, O.**, 1991. The Prediction of Hillslope Flow Paths for Distributed Hydrological Modelling Using Digital Terrain Models. *Hydrological Processes*, 5(1), 59-79.
- **Sigle, M., Hellwich, O., Köstli, A.**, 1992. Intersection and Combination of Digital Elevation Models - Methods and Applications. *International Archives of Photogrammetry and Remote Sensing*, 29(B4), 878-882.
- **Sutherland, I.E., Sproull, R.F., Schumacker, R.A.**, 1974. A Characterization of Ten Hidden-Surface Algorithms. *ACM Computing Surveys*, 6(1), 1-55.
- **Swisstopo**, 1991. Digital Terrain Model DTM/DHM25 (Reproduced with authorisation by swisstopo (BA057224)), 1:25000. Bern: Swisstopo.
  Download: http://www.swisstopo.ch

- **Wilson, J. , Gallant, J.**, 2000. *Terrain Analysis: Principles and Applications*. New York: John Wiley and Sons.
- **Yoeli, P.**, 1985. The Making of Intervisibility Maps with Computer and Plotter. *Cartographica*, 22(3), 88-103.